MATH 112 B
Final Exam
June 11, 2003

Name ______________________________

Student ID #______________________

Section ______________________

1	15	
2	20	
3	20	
4	12	
5	16	
6	17	
Total	100	

- Check that you have a complete exam. There are six problems, one on each page.
- There are multiple versions of the exam. It will be apparent if you copy someone else's work. Students found engaging in academic misconduct will receive a 0 on this exam.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- When rounding is necessary, you may round your final answer to 2 digits after the decimal.
- We can only give you credit for computations that appear on your exam. Show **all** your work.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 1 hour and 50 minutes to complete the exam.

GOOD LUCK!!

1. (15 points) Compute each of the following.

(a) $\displaystyle\int 5x - \frac{9}{2x^3} + \frac{7}{\sqrt{x}}\ dx$

(b) $\displaystyle\int_1^6 x^3 - 10x + 5\ dx$

(c) $\dfrac{dy}{dx}$ if $y = \dfrac{\sqrt{x} \cdot e^x}{\ln(3x+2)}$ (Do not simplify.)

2. (20 points) The Total Revenue and Total Cost for selling Items are given by the formulas:

$$TR(q) = \frac{1}{9}q^3 - 3q^2 + 27.3q \text{ and } TC(q) = q^2 - 11q + 47.$$

(a) Find the value of q at which Marginal Cost is \$36.

ANSWER: $q =$____________________

(b) What is the smallest possible value of Marginal Revenue?

ANSWER: \$____________________

(c) Give the longest interval on which Marginal Revenue is decreasing and Total Cost is increasing or explain why no such interval exists.

ANSWER: from $q =$____________________to $q =$____________________

(d) Find the quantity in the interval from $q = 4$ to $q = 12$ at which Profit has a global minimum.

ANSWER: $q =$____________________

3. (20 points) Mick and Rick are in their hot-air balloons. After t seconds, Mick is $M(t)$ feet off the ground and Rick is $R(t)$ feet off the ground. At time t seconds, Mick's instantaneous rate of ascent (in feet per second) is given by the function

$$m(t) = \frac{1}{8}t^2 - 3t + 16.$$

Assume that Mick's balloon is on the ground at $t = 0$ so that $M(0) = 0$.

Rick's average rate of ascent from t to $t + h$ seconds is given by

$$\frac{R(t+h) - R(t)}{h} = -\frac{1}{5}t^2 - \frac{1}{5}th - \frac{1}{15}h^2 + \frac{12}{5}t + \frac{6}{5}h - 4.$$

(a) How far off the ground is Mick after 7 seconds?

ANSWER: ____________________feet

(b) What is Mick's overall average rate of ascent after 6 seconds?

ANSWER: ____________________feet per second

(c) What is Rick's average rate of ascent from 2 to 6 seconds?

ANSWER: ____________________feet per second

(d) Find the formula for $r(t)$, Rick's instantaneous rate of ascent at time t.

ANSWER: $r(t) =$____

(e) Suppose that Rick is 57 feet off the ground at $t = 0$. Find a formula for $R(t)$, Rick's altitude at t seconds.

ANSWER: $R(t) =$____

4. (12 points) Fred's Small Appliances, Inc. makes toasters and waffle irons. Let t be the number (measured in thousands) of toasters Fred produces and sells each month and let w be the number (measured in thousands) of waffle irons Fred produces and sells each month.

   (a) Each toaster sells for \$28 and each waffle iron sells for \$31. Find the formula for $R(t, w)$, Fred's monthly Total Revenue (in thousands of dollars).

   ANSWER: $R(t, w) =$____________________________________

   (b) Fred's monthly Total Cost (in thousands of dollars) is given by the formula

$$C(t, w) = t^2 + 10.6tw + 8.9w^2 + 19.$$

   Find the formula for $P(t, w)$, Fred's monthly Profit (again in thousands of dollars).

   ANSWER: $P(t, w) =$____________________________________

   (c) Compute the partial derivatives $\frac{\partial P}{\partial t}$ and $\frac{\partial P}{\partial w}$.

   ANSWER: $\frac{\partial P}{\partial t} =$____________________________________

   $\frac{\partial P}{\partial w} =$____________________________________

   (d) Suppose Fred produces exactly $w = 1$ thousand waffle irons per month. How many toasters must Fred produce in order to maximize Profit?

   ANSWER: ____________________thousand toasters

5. (16 points) Let $u(x) = \frac{5}{6}x^4 + \frac{1}{4}x^3 - 2x^2 + x$ and define a new function $f(x)$ to be the slope of the diagonal line through the point $(x, u(x))$. Then,

$$f(x) = \frac{u(x)}{x}.$$

We also have a function $g(x)$ whose derivative is given by

$$g'(x) = -\frac{1}{2}x^2 + \frac{3}{2}x + 6.$$

(a) Find the formula for $f'(x)$.

ANSWER: $f'(x) =$________________________________

(b) Find a positive value of x at which the tangent lines to $f(x)$ and $g(x)$ are parallel.

ANSWER: $x =$____________________

(c) Suppose $g(0) = 15$. Compute $g(10)$.

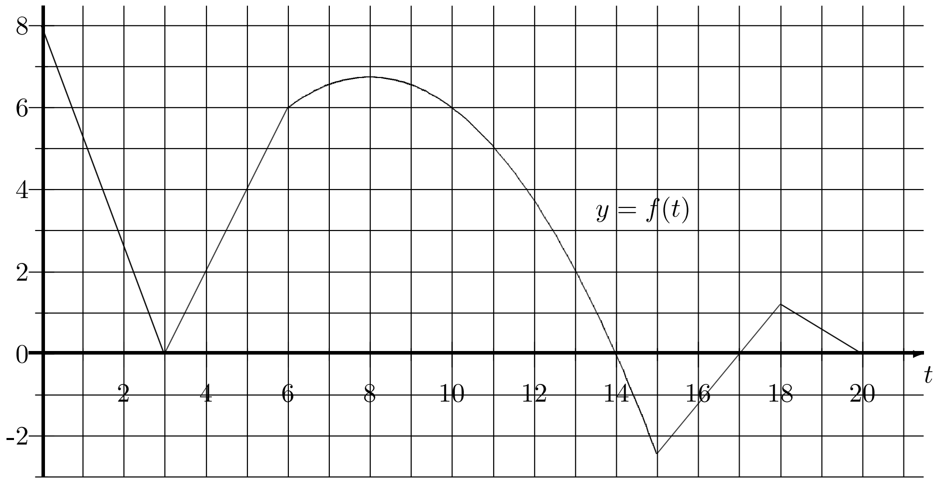
ANSWER: $g(10) =$____________________

(d) Determine whether $u(x)$ is concave up or concave down at $x = 6$. Show some work that demonstrates how you arrived at your conclusion.

ANSWER: (circle one) concave up concave down

6. (17 points) The graph below is of the function $y = f(t)$. Using it, we define another function:

$$A(m) = \int_0^m f(t)\ dt.$$



(a) Compute the approximate value of $A(5) - A(3)$.

ANSWER: $A(5) - A(3) =$____________________

(b) For how many values of m is $A'(m) = 1$?

ANSWER: ____________________values of m

(c) Compute the value of $A''(4.5)$.

ANSWER: $A''(4.5) =$____________________

(d) Name all values of m at which $A(m)$ has a horizontal tangent.

ANSWER: $m =$______________________________

(e) Indicate whether each of the following statements is True or False.

T **F** $A(17)$ is smaller than $A(14)$.

T **F** $A(10)$ is larger than $A(12)$.

T **F** $f''(8)$ is negative.

T **F** $A(m)$ has a local maximum at $m = 3$.

T **F** $A(m)$ has a local minimum at $m = 3$.