

MATH 112B
Final Exam Hints and Answers
June 11, 2003

1. (a) ANSWER: $\frac{5}{2}x^2 + \frac{9}{4}x^{-2} + 14x^{1/2} + K$
(b) ANSWER: 173.75
(c) ANSWER: $\frac{\ln(3x+2) \left[\sqrt{x}e^x + e^x \cdot \frac{1}{2\sqrt{x}} \right] - \sqrt{x} \cdot e^x \cdot \frac{3}{3x+2}}{(\ln(3x+2))^2}$
2. (a) HINT: $MC(q) = TC'(q) = 2q - 11$. Set this equal to 36 and solve for q .
ANSWER: $q = 23.5$
(b) HINT: $MR(q) = TR'(q) = \frac{1}{3}q^2 - 6q + 27.3$. This is a quadratic whose graph is a parabola that opens upward. Its vertex is $(9, 0.30)$.
ANSWER: \$0.30
(c) HINT: $TC(q)$ is a quadratic whose graph is a parabola that opens upward. Its vertex occurs at $q = 5.5$. TC increases from 5.5 on and MR decreases from 0 to 9.
ANSWER: from $q = 5.5$ to $q = 9$
(d) HINT: $P(q) = \frac{1}{9}q^3 - 4q^2 + 38.3q - 47$ and $P'(q) = \frac{1}{3}q^2 - 8q + 38.3$. Use the quadratic formula to find that $P'(q) = 0$ at $q = 6.61$ and $q = 17.39$. Compute $P(4)$, $P(6.61)$ and $P(12)$ and select the smallest one.
ANSWER: $q = 12$
3. (a) HINT: $M(t) = \frac{1}{24}t^3 - \frac{3}{2}t^2 + 16t + K$. $M(0) = K$ and $M(0) = 0$. So, $K = 0$. Compute $M(7)$.
ANSWER: 52.79 feet
(b) HINT: Compute $\frac{M(6)}{6}$.
ANSWER: 8.5 feet per second
(c) HINT: Let $t = 2$ and $h = 4$ and use the formula given for Rick's average rate of ascent.
ANSWER: 2.13 feet per second
(d) HINT: Let h go to zero in the formula for Rick's average rate of ascent.
ANSWER: $r(t) = -\frac{1}{5}t^2 + \frac{12}{5}t - 4$
(e) HINT: $R(t) = -\frac{1}{15}t^3 + \frac{6}{5}t^2 - 4t + K$. $R(0) = K$ and $R(0) = 57$. So, $K = 57$.
ANSWER: $R(t) = -\frac{1}{15}t^3 + \frac{6}{5}t^2 - 4t + 57$
4. (a) ANSWER: $R(t, w) = 28t + 31w$
(b) ANSWER: $P(t, w) = 28t + 31w - t^2 - 10.6tw - 8.9w^2 - 19$
(c) ANSWER: $\frac{\partial P}{\partial t} = 28 - 2t - 10.6w$ and $\frac{\partial P}{\partial w} = 31 - 10.6t - 17.8w$
(d) HINT: Let $w = 1$ in the formula for profit. Then you have $P(t, 1) = -t^2 + 17.4t + 3.1$, which is a quadratic function with variable t . The maximum profit will occur at its vertex.
ANSWER: 8.7 thousand toasters
5. (a) HINT: $f(x) = \frac{5}{6}x^3 + \frac{1}{4}x^2 - 2x + 1$
ANSWER: $f'(x) = \frac{5}{2}x^2 + \frac{1}{2}x - 2$
(b) HINT: Set $f'(x) = g'(x)$ and solve for x .
ANSWER: $x = 1.81$
(c) HINT: $g(x) = -\frac{1}{6}x^3 + \frac{3}{4}x^2 + 6x + K$. $g(0) = K$ and $g(0) = 15$. So, $K = 15$.
ANSWER: $g(10) = -16.67$
(d) HINT: $u''(x) = 10x^2 + \frac{3}{2}x - 4$. Compute $u''(6)$.
ANSWER: concave up

6. (a) HINT: Count squares or use the triangle under the graph from $t = 3$ to $t = 5$.
ANSWER: $A(5) - A(3) = 4$ (We accepted any answer between 3.5 and 4.5.)
- (b) HINT: A' is the same as f .
ANSWER: 5 values of m
- (c) HINT: Since A' is the same as f , A'' is the same as f' . The graph given is the graph of f . So, $A''(4.5) = f'(4.5)$ is the slope of the tangent line to f at $t = 4.5$.
ANSWER: 2
- (d) HINT: A has a horizontal tangent whenever the graph of A' (which is the same as f) crosses the t -axis.
ANSWER: $m = 3, 14, 17, 20$
- (e) ANSWERS: T F T F F