MATH 112
Final Exam
June 5, 2004

Name _____

Student ID #_____

Section _____

1	16	
2	16	
3	17	
4	18	
5	16	
6	17	
Total	100	

- Check that you have six problems in your exam packet.
- There are multiple versions of the exam. It will be apparent if you copy someone else's work. Students found engaging in academic misconduct will receive a 0 on this exam and will likely fail the course. Please do not cheat.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- When rounding is necessary, you may round your final answer to 2 digits after the decimal.
- We can only give you credit for computations that appear on your exam. Show **all** your work.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 3 hours to complete the exam.

GOOD LUCK!

1. (16 points) Compute each of the following.

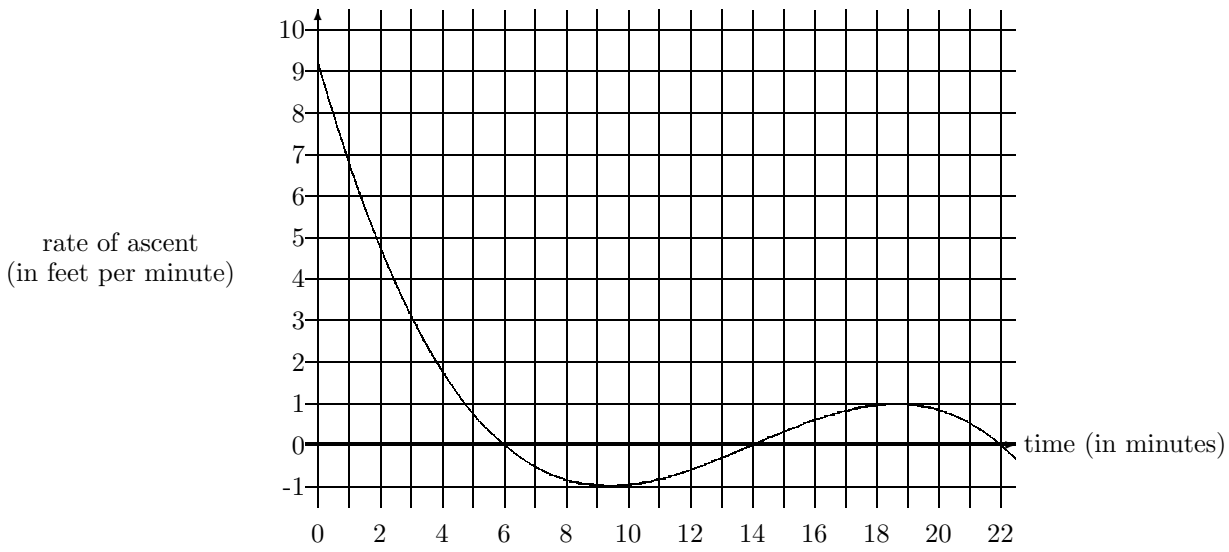
(a) $\displaystyle\int \frac{1}{8}z^{1/8} - 4 + \frac{1}{z^{1/6}}\ dz$

(b) $\displaystyle\int_3^6 x^2 + 4x + 12\ dx$

(c) $\dfrac{dy}{dx}$ if $y = \dfrac{\frac{1}{x} - \frac{1}{x^2}}{7x + e^x}$ (Do not simplify.)

(d) $\dfrac{\partial z}{\partial t}$ if $z = \sqrt{\ln(rt^2 + 70r^4t)}$ (Do not simplify.)

2. (16 points) Below is the graph of the instantaneous rate of ascent, $r(t)$, of a hot-air balloon. Let $A(t)$ be the altitude of the balloon, in feet, after t minutes.



(a) List all values of t at which the graph of $A(t)$ has a horizontal tangent.

ANSWER: $t =$ _____

(b) List all values of t at which the graph of $A''(t)$ crosses the t-axis.

ANSWER: $t =$ _____

(c) Give a value of m such that $A'(m) = 6$.

ANSWER: $m =$ _____

(d) Suppose $A(3) = 50$ feet. What was the balloon's altitude at $t = 0$?

ANSWER: _____ feet

(e) Is the graph of $A(t)$ concave up or concave down at $t = 16$?

ANSWER: (circle one) UP DOWN

(f) Use the graph to estimate the value of $\int_{13}^{15} r(t)\ dt$.

ANSWER: $\int_{13}^{15} r(t)\ dt =$ _____

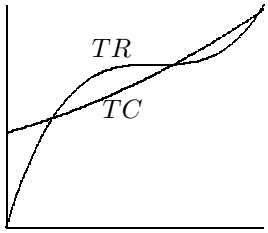
3. (17 points)

You sell Items. The total revenue and total cost for selling q hundred Items are:

$$TR(q) = 4q^3 - 84q^2 + 588q \text{ hundred dollars}$$

$$TC(q) = 3q^2 + 50q + 30 \text{ hundred dollars.}$$

The graphs of TR and TC are given at right.



(a) Find the largest quantity at which marginal revenue is equal to \$108.

ANSWER: $q =$_____hundred Items

(b) Average revenue is given by $AR(q) = \dfrac{TR(q)}{q}$. Find a positive value of q at which average revenue is equal to marginal revenue.

ANSWER: $q =$_____hundred Items

(c) Compute the cost of the 801^{st} Item.

ANSWER: _____dollars

(d) Give the longest interval on which total cost is decreasing and marginal revenue is decreasing. If no such interval exists, explain why.

ANSWER: from $q =$_____to $q =$_____

(e) Give the longest interval on which marginal revenue is decreasing and concave up. If no such interval exists, explain why.

ANSWER: from $q =$_____to $q =$_____

4. (18 points) Mary Kate and Ashley are each pouring water into a vat. After t minues, Mary Kate's vat contains $M(t)$ gallons and Ashley's contains $A(t)$ gallons.

   At time t minutes, Mary Kate is pouring water into her vat at an instantaneous rate of

   $$m(t) = 0.5t \text{ gallons per minute.}$$

   At $t = 0$, Mary Kate's vat contains 10 gallons.

   From time t to time $t + h$ minutes, Ashley pours at an average rate of

   $$\frac{A(t+h) - A(t)}{h} = \frac{1}{(t+5)(t+h+5)} \text{ gallons per minute.}$$

   (a) How much water is in Mary Kate's vat after 12 minutes?

   ANSWER: _____gallons

   (b) What is the overall rate of flow into Mary Kate's vat over the first four minutes?

   ANSWER: _____gallon(s) per minute

   (c) What is the average rate of flow into Ashley's vat from $t = 1$ to $t = 7$ minutes?

   ANSWER: _____gallons per minute

   (d) Find a formula for $a(t)$, the instantaneous rate of flow into Ashley's vat at time t.

   ANSWER: $a(t) =$_____

   (e) Suppose there are 23.8 gallons in Ashley's vat at $t = 0$. Find the formula for $A(t)$, the amount in Ashley's vat at time t.

   ANSWER: $A(t) =$_____

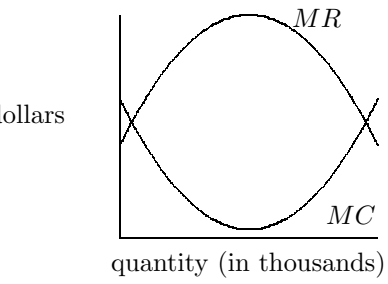
5. (16 points)

You sell Things. The marginal revenue and marginal cost for selling q thousand Things are:

$$MR(q) = -q^2 + 15q + 40 \text{ dollars}$$

$$MC(q) = q^2 - 15q + 60 \text{ dollars.}$$

The graphs of MR and MC are given at right.



(a) Find the formula for variable cost at q thousand Things.

ANSWER: $VC(q) =$ _____

(b) The total cost to produce 12 thousand Things is 372 thousand dollars. Find the formula for total cost to produce q thousand Things.

ANSWER: $TC(q) =$ _____

(c) Find the quantity that will yield the largest profit.

ANSWER: $q =$ _____ thousand Things

(d) What is the largest possible profit?

ANSWER: _____ thousand dollars

6. (17 points) The value of a stock you're keeping your eye on appears to grow exponentially. You wish to find an exponential model to predict $V(t)$, the value of one share of the stock in dollars, t days after your observation begins. As a good Math 112 student, you know that the way to do this is to take natural logarithms and find $z = mt + b$, the line of best fit for the logarithmic data. You compute the mean-squared error function for the logarithmic data:

$$E(b, m) = b^2 + 42m^2 + 12bm - 3.12b - 20.46m + 2.5602.$$

(a) Compute the partial derivatives $\dfrac{\partial E}{\partial b}$ and $\dfrac{\partial E}{\partial m}$.

ANSWERS: $\dfrac{\partial E}{\partial b} =$_____

$\dfrac{\partial E}{\partial m} =$_____

(b) Find the line of best-fit: $z = mt + b$.

ANSWER: _____

(c) What is the smallest possible value of $E(b, m)$? (Do not round your answer.)

ANSWER: _____

(d) Compute the value of one share your stock in dollars, 11 days after your observation begins. (Round to the nearest cent.)

ANSWER: \$_____