MATH 112 - Spring 2004
Final Exam - Hints and Answers

1. (a) $\dfrac{1}{9}z^{9/8} - 4z + \dfrac{6}{5}z^{5/6} + C$
   (b) 153
   (c) $\dfrac{dy}{dx} = \dfrac{(7x+e^x)(-x^{-2}+2x^{-3}) - (\frac{1}{x}-\frac{1}{x^2})(7+e^x)}{(7x+e^x)^2}$
   (d) $\dfrac{\partial z}{\partial t} = \dfrac{1}{2}[\ln(rt^2+70r^4t)]^{-1/2} \cdot \dfrac{1}{rt^2+70r^4t} \cdot (2rt+70r^4)$

2. NOTE: The graph given in this problem is the graph of $A'(t)$.

   (a) HINT: $A(t)$ has horizontal tangents whenever $A'(t)$ crosses the t-axis.
   ANSWER: $t = 6$, 14, 22
   (b) HINT: $A''(t)$ crosses the t-axis whenever $A'(t)$ has a horizontal tangent.
   ANSWER: $t \approx 9.2$, 18.8
   (c) HINT: Find where $A'(m) = 6$.
   ANSWER: $m \approx 1.3$
   (d) HINT: $A(3) - A(0)$ is the area under the rate graph from $t = 0$ to $t = 3$. Count squares or use a trapezoid to compute that $A(3) - A(0) = 18$. They tell you that $A(3) = 50$. So, $50 - A(0) = 18$.
   ANSWER: 32 feet
   (e) HINT: You need to know the sign of $A''(16)$. You have the graph of $A'(t)$. So, $A''(16)$ is the slope of the tangent line to the given graph at $t = 16$.
   ANSWER: UP
   (f) ANSWER: 0

3. (a) HINT: $MR(q) = 12q^2 - 168q + 588$. Set $MR(q) = 108$ and solve for q.
   ANSWER: 10 hundred Items
   (b) HINT: $AR(q) = 4q^2 - 84q + 588$. Set $AR(q) = MR(q)$ and solve for q.
   ANSWER: 10.5 hundred Items
   (c) HINT: $MC(q) = 6q + 50$. Compute $MC(8)$.
   ANSWER: 98 dollars
   (d) ANSWER: TC is always increasing; so no interval exists.
   (e) HINT: $MR(q) = 12q^2 - 168q + 588$ is a quadratic whose graph is a parabola that opens upward. Its vertex is at $q = 7$. So, MR is decreasing from $q = 0$ to $q = 7$. Further, $MR''(q) = 24$, which is positive for all values of q. Therefore, MR is always concave up.
   ANSWER: from $q = 0$ to $q = 7$

4. (a) HINT: Anti-differentiate $m(t)$ to get a formula for $M(t)$, the amount in Mary Kate's vat at time t: $M(t) = 0.25t^2 + C$. $M(0) = C$, but you also know that $M(0) = 10$. So, the constant C must equal 10. That gives $M(t) = 0.25t^2 + 10$. Compute $M(12)$.

   ANSWER: 46 gallons

   (b) HINT: Compute $\frac{M(4)}{4}$.

   ANSWER: 3.5 gallons per minute

   (c) HINT: Take $t = 1$ and $h = 6$ in the formula for Ashley's average rate to compute $\frac{A(7)-A(1)}{6}$.

   ANSWER: $\frac{1}{72}$ gallons per minute

   (d) HINT: Let h go to 0 in the formula for Ashley's average rate.

   ANSWER: $\frac{1}{(t+5)^2}$

   (e) HINT: There are two ways to do this problem.

   Method I : Antidifferentiate the formula you got in part (d) for the instantaneous rate of change to get

$$A(t) = \frac{-1}{t+5} + C.$$

   Then, $A(0) = -\frac{1}{5} + C$; but you also know that $A(0) = 23.8$. So, $-\frac{1}{5} + C = 23.8$ and $C = 24$.

   ANSWER: $A(t) = 24 - \frac{1}{t+5}$

   Method II : Use the formula for $\frac{A(t+h)-A(t)}{h}$ to find the formula for $A(t)-A(0)$. (You'd take $t = 0$ and $h = t$ in that formula. Is that confusing enough for you?) You would get $A(t) - A(0) = \frac{t}{5(t+5)}$. Again, you know that $A(0) = 23.8$.

   ANSWER: $A(t) = \frac{t}{5(t+5)} + 23.8$

   NOTE: These two methods give answers that look different, but amazingly enough, if you simplified these answers, they would look the same.

5. (a) HINT: $VC(q)$ is an anti-derivative of $MC(q)$: $VC(q) = \frac{1}{3}q^3 - \frac{15}{2}q^2 + 60q + K$. $VC(0) = K$ but $VC(0)$ also equals 0. So, the constant K must be 0.

   ANSWER: $VC(q) = \frac{1}{3}q^3 - \frac{15}{2}q^2 + 60q$

   (b) HINT: $TC(q) = VC(q) + FC$. You have the formula for VC, so you just need FC. $TC(12) = VC(12) + FC$. Evaluate VC at $q = 12$, substitute in 372 for $TC(12)$ and solve for FC.

   ANSWER: $TC(q) = \frac{1}{3}q^3 - \frac{15}{2}q^2 + 60q + 156$

   (c) HINT: Set $MR = MC$ and solve for q. You get two answers. One is where profit is maximized; the other is where profit has a local minimum.

   ANSWER: $q = 14.3$ thousand Things

(d) HINT: You'll need the formula for Profit $(TR(q) - TC(q))$. You already have a formula for $TC(q)$ from part (b). Antidifferentiate $MR(q)$ to find a formula for $TR(q)$. Plug $q = 14.3$ into the profit function.

ANSWER: 675.88 thousand dollars

6. (a) ANSWER: $\frac{\partial E}{\partial b} = 2b + 12m - 3.12$; $\frac{\partial E}{\partial m} = 84m + 12b - 20.46$

(b) ANSWER: $z = 0.145t + 0.69$

(c) HINT: The m and b that give the best-fitting line give the global minimum value of $E(b, m)$. So, evaluate $E(0.69, 0.145)$.

ANSWER: 0.00045

(d) HINT: $\ln V(t) = 0.145t + 0.69$. Solve for $V(t)$ and evaluate at $t = 11$.

ANSWER: \$9.83