

MATH 112
Final Exam
June 4, 2005

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	12	
2	10	
3	12	
4	10	
5	20	
6	16	
7	20	
Total	100	

- Check that your exam contains seven problems on nine pages.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (12 points) Compute the indicated derivative. Do not simplify.

(a) $y = \frac{\ln(3x^2 + 4)}{8x^6}$

$$\frac{dy}{dx} =$$

(b) $f(x, y) = xy^2e^{-10y}$

$$f_x(x, y) =$$

$$f_y(x, y) =$$

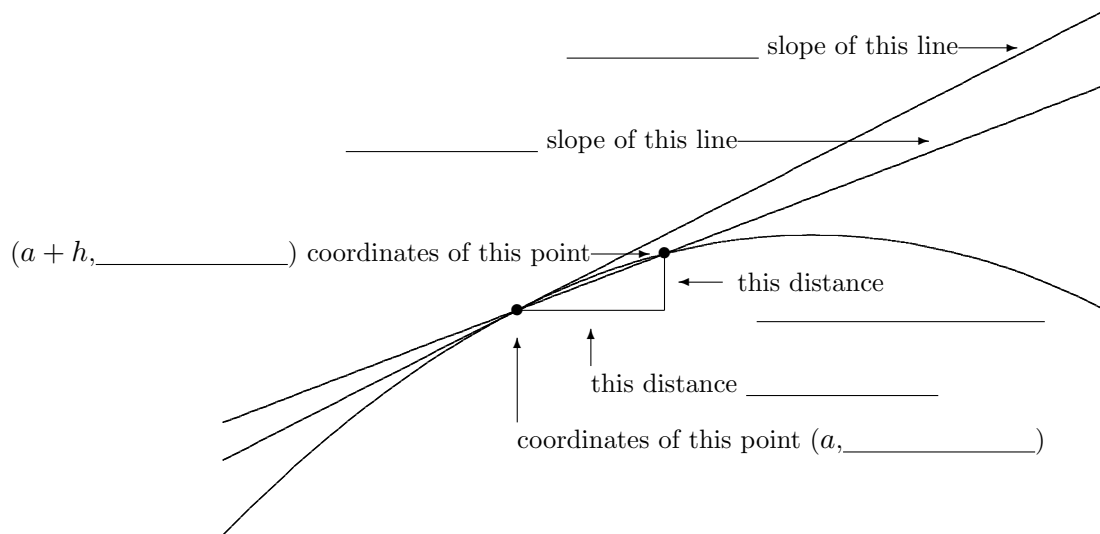
2. (10 points) Evaluate the integral.

(a) $\int \left(6x^2 - \frac{4}{x^3} + \frac{2}{\sqrt{x}} \right) dx$

(b) $\int_1^4 \left(2t - \frac{9t^2}{4} - \frac{t^{3/2}}{2} \right) dt$

3. (12 points) Below is a blow-up of the graph of $f(x)$ near $x = a$, along with the tangent line to $f(x)$ at $x = a$ and a secant line from $x = a$ to $x = a + h$. Each of the following expressions belongs in one of the blanks below. Put each item in the correct blank.

- I. $f(a)$
- II. $f'(a)$
- III. $f(a + h)$
- IV. h
- V. $f(a + h) - f(a)$
- VI. $\frac{f(a + h) - f(a)}{h}$



4. (10 points) Pete's Pink Paints Shop offers two shades: Blush and Bashful. Each shade is made by mixing white paint with red paint. Blush is 75% white and 25% red; Bashful is 50% white and 50% red.

Each week, Pete receives 800 gallons of white paint and 600 gallons of red paint from his supplier. He earns \$6 profit on each gallon of Blush and \$4.75 profit on each gallon of Bashful.

Let x be the number of gallons of Blush and y be the number of gallons of Bashful Pete produces and sells each week.

- (a) Give a formula for $W(x, y)$, the number of gallons of white paint Pete needs in order to make x gallons of Blush and y gallons of Bashful.

$$W(x, y) = \underline{\hspace{10cm}}$$

- (b) Give a formula for $R(x, y)$, the number of gallons of red paint Pete needs in order to make x gallons of Blush and y gallons of Bashful.

$$R(x, y) = \underline{\hspace{10cm}}$$

- (c) How many gallons of each shade should Pete produce in order to maximize profit? Use the method of linear programming and show all your work.

ANSWER: Blush: $x = \underline{\hspace{10cm}}$ gallons

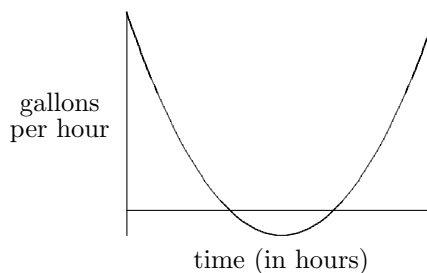
Bashful: $y = \underline{\hspace{10cm}}$ gallons

5. (20 points)

Water is flowing in and out of two vats. The *rate of flow* for Vat A is given by the formula

$$a(t) = 3t^2 - 36t + 96,$$

which is graphed at right.



The *amount* in Vat B is given by the formula

$$B(t) = -t^3 + 55.5t^2 + 97.$$

- (a) The *amount* in Vat A is given by the function $A(t)$. On the interval from $t = 0$ to $t = 10$, find all values of t at which $A(t)$ has a local optimum. Tell whether each is a local maximum or a local minimum.

ANSWERS: (you may not use all of the blanks)

$t =$ _____ gives a (circle one) local max local min

$t =$ _____ gives a (circle one) local max local min

$t =$ _____ gives a (circle one) local max local min

$t =$ _____ gives a (circle one) local max local min

- (b) How much water flows into Vat A during the interval from $t = 2$ to $t = 4$ hours?

ANSWER: _____ gallons

(This problem is continued on the next page.)

Again, the rate of flow for Vat A is

$$a(t) = 3t^2 - 36t + 96$$

and the amount in Vat B is

$$B(t) = -t^3 + 55.5t^2 + 97.$$

- (c) At $t = 0$, Vat A contains 25 gallons more than Vat B. Write out a formula for the amount in Vat A.

ANSWER: $A(t) =$ _____

- (d) At what time is the level in Vat B rising most rapidly? (That is, at what time is the rate of flow into Vat B largest?)

ANSWER: $t =$ _____ hours

- (e) What is the highest level the water reaches in Vat B during the interval from $t = 0$ to $t = 50$?

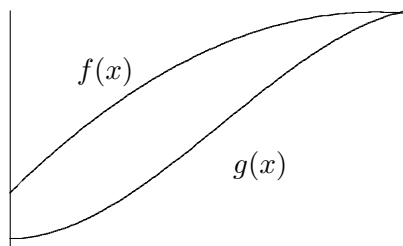
ANSWER: _____ gallons

6. (16 points)

To the right are rough sketches of two functions:

$$f(x) = -5x^2 + 82x + 100 \text{ and}$$

$$g(x) = -x^3 + 14.25x^2 + 15.$$



(a) Find a value of x at which the slope of the tangent line to $f(x)$ is 66.

ANSWER: $x =$ _____

(b) Define a new function $h(x)$ by $h(x) = \frac{g(x)}{x}$. Is $h(x)$ concave up or down at $x = 2$?

ANSWER: (circle one) concave up concave down

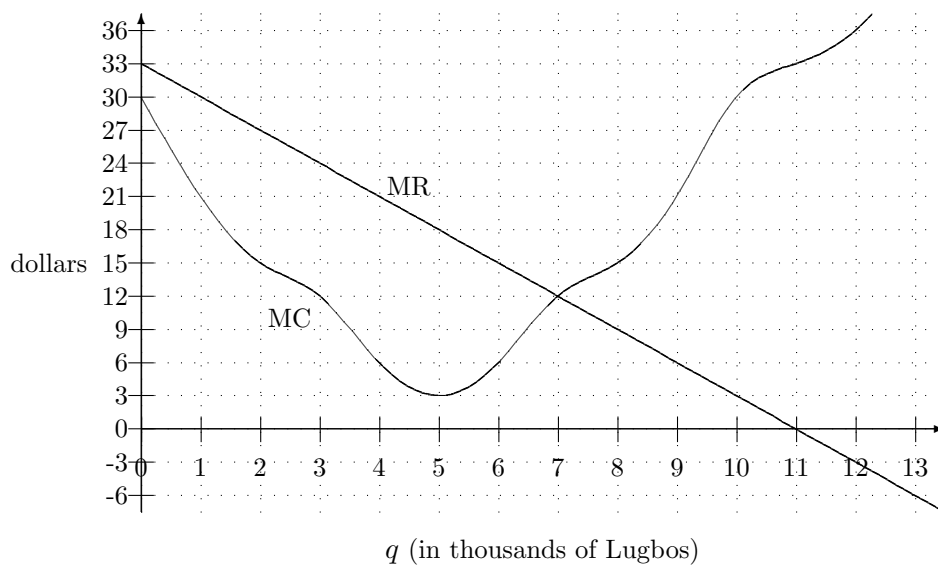
(c) Let $D(x) = f(x) - g(x)$. Find the smallest value of x at which the graph of $D(x)$ has a horizontal tangent.

ANSWER: $x =$ _____

(d) Use the Second Derivative Test to determine whether $D(x)$ has a local maximum or a local minimum at the value you found in part (c).

ANSWER: (circle one) local max local min

7. (20 points) The graphs of MR and MC for selling Lugbos are given below.



- (a) Determine the change in total cost (TC) that results from increasing quantity from $q = 6$ thousand Lugbos to $q = 8$ thousand Lugbos. Is this an increase or a decrease in TC ?

ANSWER: Total Cost (circle one) increases decreases

by _____ thousand dollars

- (b) What quantity gives the largest value of total revenue (TR)? Show some work or explain how you know your answer is correct.

ANSWER: $q =$ _____ thousand Lugbos
(This problem is continued on the next page.)

(c) Complete the following table.

interval	0–1	1–2	2–3	3–4	4–5	5–6	6–7
area between MR and MC on that interval	6	10.5			15	12	

(d) Determine the change in profit that results from increasing quantity from $q = 3$ thousand Lugbos to $q = 6$ thousand Lugbos. Is this an increase or a decrease in profit?

ANSWER: Profit (circle one) increases decreases

by _____ thousand dollars

(e) Fixed cost is \$16,500. That is, $FC = 16.5$ thousand dollars. Name the smallest quantity at which profit is equal to 0.

ANSWER: $q =$ _____ thousand Lugbos

(f) What is the largest possible profit? (Again, $FC = 16.5$.)

ANSWER: _____ thousand dollars