MATH 112 – FINAL EXAM Hints and Answers
Spring 2005

1. (a) ANSWER: $\dfrac{dy}{dx} = \dfrac{8x^6 \cdot \frac{1}{3x^2+4} \cdot 6x - \ln(3x^2+4) \cdot 48x^5}{(8x^6)^2}$

   (b) ANSWER: $f_x(x,y) = y^2 e^{-10y}$; $f_y(x,y) = x\left[y^2 \cdot e^{-10y} \cdot (-10) + e^{-10y} \cdot 2y\right]$

2. (a) ANSWER: $2x^3 + \frac{2}{x^2} + 4\sqrt{x} + K$

   (b) ANSWER: -38.45

3. ANSWER: Starting with the far left blank and moving clockwise, the blanks are filled in as follows: III, VI, II, V, IV, and I.

4. (a) ANSWER: $W(x,y) = 0.75x + 0.50y$

   (b) ANSWER: $R(x,y) = 0.25x + 0.50y$

   (c) HINT: The vertices of the feasible region are $(0,0)$, $(0,1200)$, $(1066.67,0)$, and $(400,1000)$.
   ANSWER: $x = 400$ and $y = 1000$

5. (a) HINT: $A'(t) = a(t)$, the formula you are given. Set $a(t)$ equal to zero and solve for t. Then use the picture to see where $a(t)$ is changing from negative to positive (which gives $A(t)$ a minimum) and where $a(t)$ is changing from positive to negative (which gives $A(t)$ a maximum).
   ANSWER: $t = 4$ gives a local maximum; $t = 8$ gives a local minimum

   (b) HINT: You're looking for $A(4) - A(2)$ which is equal to $\int_2^4 a(t)\,dt$.
   ANSWER: 32 gallons

   (c) HINT: $A(t)$ is an anti-derivative of $a(t)$: $A(t) = t^3 - 18t^2 + 96t + K$, for some constant K. We know that $A(0) = 25 + B(0) = 25 + 97 = 122$. But $A(0)$ also equals K. So, $K = 122$.
   ANSWER: $A(t) = t^3 - 18t^2 + 96t + 122$

   (d) HINT: We want to know when the function $B'(t)$ is largest. $B'(t)$ is a quadratic function whose graph is a parabola that opens downward. It is largest at its vertex.
   ANSWER: $t = 18.5$

   (e) HINT: Find the critical numbers of $B(t)$ by setting $B'(t)$ equal to 0 and solving for t. (The critical numbers are $t = 0$ and $t = 37$.) Plug the critical numbers and endpoints into $B(t)$ and choose the largest value.
   ANSWER: 25,423.5 gallons

6. (a) HINT: Find the derivative of $f(x)$, set $f'(x)$ equal to 66, and solve for x.
   ANSWER: $x = 1.6$

   (b) HINT: $h(x) = -x^2 + 14.25x + \frac{15}{x}$, $h'(x) = -2x + 14.25 - \frac{15}{x^2}$, $h''(x) = -2 + \frac{30}{x^3}$. Determine whether $h''(2)$ is positive or negative.
   ANSWER: concave up

   (c) HINT: Compute $D(x)$ and its derivative. Set $D'(x)$ equal to 0 and solve for x. You will get two solutions. Choose the smallest.
   ANSWER: $x = 2.70$

   (d) HINT: $D''(x) = 6x - 38.5$
   ANSWER: local max

7. (a) HINT: Compute the area under MC from $q = 6$ to $q = 8$. Since this region is above the q-axis, the result is an increase in TC.

   ANSWER: TC increases by 22.5 thousand dollars

   (b) HINT: MR is positive for a while and then becomes negative. Since MR is the derivative of TR, this means that TR increases for a while and then starts to decrease. TR is largest when MR is crossing the q-axis.

   ANSWER: $q = 11$ thousand Lugbos

   (c) ANSWER: 2–3: 12; 3–4: 13.5; 6–7: 4.5

   (d) HINT: From $q = 3$ to $q = 6$, MR is greater than MC. This means profit is increasing on this entire interval and that the change in profit is equal to the area of the region between MR and MC on the interval from $q = 3$ to $q = 6$.

   ANSWER: Profit increases by 40.5 thousand dollars.

   (e) HINT: You need to find the smallest value of q such that the area between MR and MC from 0 to that q is equal to 16.5.

   ANSWER: $q = 2$ thousand Lugbos

   (f) HINT: Profit is maximized where MR is equal to MC. This is at $q = 7$. So, maximum profit is equal to the area between MR and MC from $q = 0$ to $q = 7$, minus the value of the fixed cost.

   ANSWER: 57 thousand dollars