

MATH 112
Final Exam
June 3, 2006

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	16	
2	15	
3	10	
4	12	
5	10	
6	14	
7	13	
8	10	
Total	100	

- Check that your exam contains 8 problems.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method or read a value from a graph on your calculator when an algebraic method is available, you may not receive full credit.

GOOD LUCK!

1. (16 points) Find the indicated derivatives. Do not simplify.

(a) $f(x) = (4 + 3x)(\sqrt[3]{x} - 5x^2)$

$$f'(x) =$$

(b) $g(t) = \ln(t^3 - 3t)$

$$g'(t) =$$

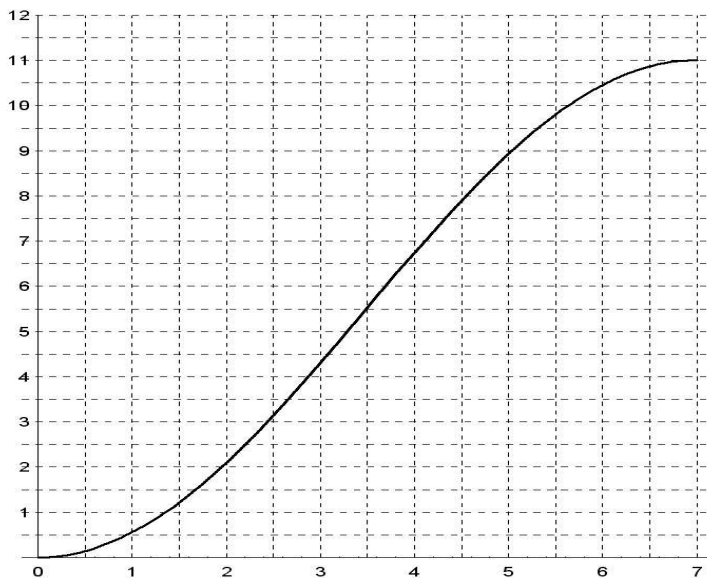
(c) $y = \frac{xe^x}{1 - x^3}$

$$\frac{dy}{dx} =$$

(d) $z = e^x y + x + y^2 - \frac{3}{xy}$

$$\frac{\partial z}{\partial x} =$$

2. (15 points) The graph below is the graph of the Total Revenue $TR(q)$ for manufacturing and selling Items. The quantity q is given in **thousands of Items**, and the function $TR(q)$ in **thousands of dollars**. Recall that the Average Revenue from selling q thousand Items is $AR(q) = \frac{TR(q)}{q}$.



Use this graph and the methods studied in this class to answer the following questions. Be as precise as possible, and indicate clearly how you get your answers. Draw and label on the graphs provided any lines or distances you use.

For parts (a) and (b), **INCLUDE UNITS WITH YOUR ANSWERS.**

- (a) Estimate your Average Revenue if you sell 6 thousand Items.

ANSWER: _____

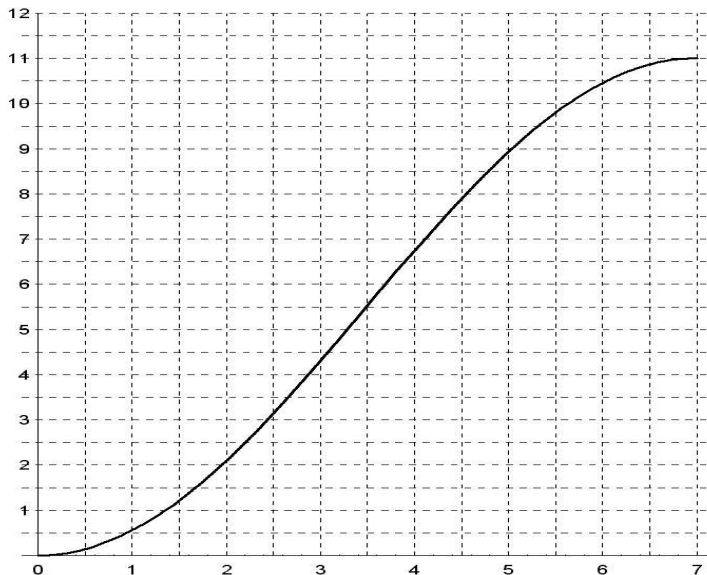
- (b) Estimate the extra revenue you get from selling the 6001st Item, if you have already sold 6000 Items.

ANSWER: _____

- (c) Find a range of values for q over which the Total Revenue is increasing, but the Marginal Revenue is decreasing.

ANSWER: from $q =$ _____ to $q =$ _____ thousand Items

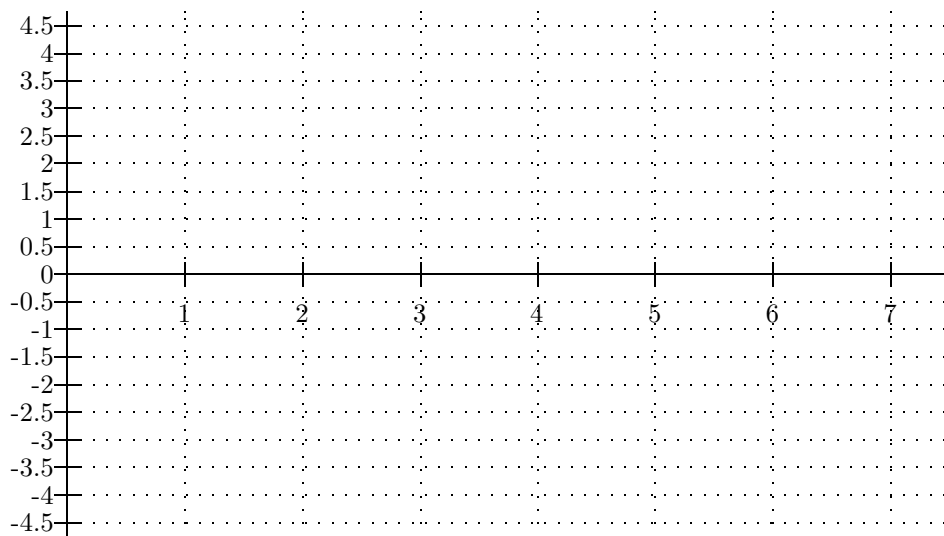
Here's the graph of TR again:



- (d) Find two quantities q where $MR(q) = \$2$ per Item.

ANSWER: $q =$ _____ and _____ thousand Items.

- (e) Sketch the graph of the Marginal Revenue function *vs.* quantity q . Label the coordinates of the q -intercept(s) and any local optima of Marginal Revenue. No need to explain your answer in this part.



3. (10 points)

A car travels for five minutes on a straight road. The distance (in miles) the car travels from the beginning of the trip to time t minutes is given by the function $d(t) = -\frac{1}{5}t^2 + 2t$.

- (a) Write a formula in terms of h for the distance covered by the car from $t = 3$ to $t = 3 + h$. Simplify as much as possible.

ANSWER: _____

- (b) Write a formula in terms of h for the average speed of the car over the h minute interval starting at $t = 3$ minutes.

ANSWER: _____

- (c) Use your answer to part (b) to compute the instantaneous speed of the car at $t = 3$ minutes.

ANSWER: _____ miles per minute

4. (12 points) You run a business selling Lollipops. Your profit from selling q thousand Lollipops is given by the function: $P(q) = q^4 - 32q^3 + 270q^2 - 200$ dollars.

You must use calculus and show all your work. Reading a value from a graph on your calculator will not earn you full credit.

- (a) Find all the critical numbers of the profit function.

ANSWER: (list all critical numbers) $q =$ _____ thousand Lollipops

- (b) Find the minimum and the maximum profit you can make if you sell between 1 thousand and 10 thousand Lollipops.

ANSWER: Min profit is \$ _____ at $q =$ _____ thousand Lollipops.

Max profit is \$ _____ at $q =$ _____ thousand Lollipops.

- (c) Find the longest interval on which the profit function is concave down.

ANSWER: from $q =$ _____ to $q =$ _____ thousand Lollipops

5. (10 points) A used-car dealership estimates that its weekly revenue in dollars is given by a function

$$W(p, r) = 100p - 45pr - 0.0025p^2,$$

where p is the average list price (**in dollars**) of all cars on the lot and r is the current interest rate (**expressed as a decimal**) being offered by the lot's associated lender.

- (a) Estimate the lot's weekly revenue if the average list price is \$12,400 and the current interest rate is 6.2%.

ANSWER: \$ _____

- (b) Compute $W_p(p, r)$ and $W_r(p, r)$.

ANSWER: $W_p(p, r) =$ _____

$W_r(p, r) =$ _____

- (c) During one week, the average list price is \$15,000 and the interest rate is 5.4%. Use a partial derivative to estimate the change in weekly revenue if, the following week, the average list price has increased to \$15,001 and the interest rate remains 5.4%.

ANSWER: \$ _____

6. (14 points) Suppose that, in order to achieve monthly sales of q thousand Things, you must sell them at a price of

$$p = h(q) = q^2 - 92q + 2116 \text{ dollars per Thing.}$$

- (a) Give the formulas for total revenue and marginal revenue for selling q thousand things.

ANSWER: $TR(q) =$ _____

$MR(q) =$ _____

- (b) Find the quantity between $q = 1$ thousand and $q = 20$ thousand Things at which the total revenue function has a critical number. Use the Second Derivative Test to determine whether this gives a local maximum or a local minimum of TR .

ANSWER: the critical number $q =$ _____

gives a (circle one) local max local min

- (c) The formula for marginal cost is given by

$$MC(q) = 3q^2 - 48q + 192.$$

If you sell $q = 28$ thousand Things, then you will break even (that is, $TR(28) = TC(28)$). What is the value of your fixed costs?

ANSWER: $FC =$ _____ thousand dollars

7. (13 points)

(a) Compute the following antiderivatives.

i.
$$\int \left(4\sqrt{t} - 5t^2 + \frac{9}{t^{2/3}} \right) dt$$

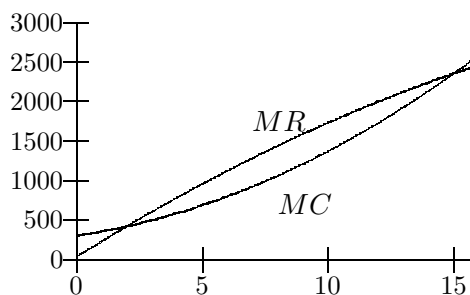
ii.
$$\int x(3x^2 - 8x + 9) dx$$

(b)

The marginal revenue and marginal cost (both in dollars) for selling q items are given by

$$MR(q) = -3q^2 + 200q + 30 \text{ and}$$

$$MC(q) = 6q^2 + 47q + 300.$$

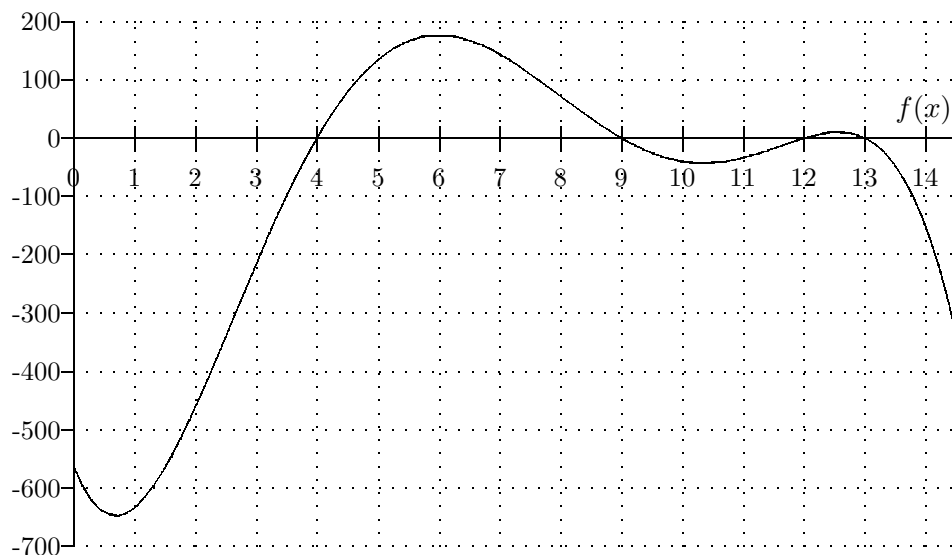


Use a definite integral to compute the change in profit from $q = 5$ to $q = 7$.

ANSWER: _____dollars

8. (10 points) Below is the graph of the function $f(x)$. We define a new function

$$F(m) = \int_0^m f(x) dx.$$



- (a) Name all values of m in the interval from $m = 0$ to $m = 14$ at which $F(m)$ has a local **maximum**. You need not justify your answer.

ANSWER: $m =$ _____

- (b) Use the graph to approximate the values of $F'(7)$ and $F''(1)$. Show your work.

ANSWER: $F'(7) =$ _____; $F''(1) =$ _____

- (c) Use the graph to approximate the value of $\int_6^8 f(x) dx$. Show your work.

ANSWER: $\int_6^8 f(x) dx =$ _____

- (d) On the interval from $m = 0$ to $m = 3$, what value of m gives the global **minimum** value of $F(m)$? You need not justify your answer.

ANSWER: $m =$ _____