

MATH 112 – FINAL EXAM Hints and Answers
Version Alpha
Spring 2006

1. (a) ANSWER: $f'(x) = 3(\sqrt[3]{x} - 5x^2) + (4 + 3x)\left(\frac{1}{3}x^{-2/3} - 10x\right)$
(b) ANSWER: $g'(t) = \frac{1}{t^3-3t}(3t^2 - 3)$
(c) ANSWER: $\frac{dy}{dx} = \frac{(e^x + xe^x)(1-x^3) - xe^x(-3x^2)}{(1-x^3)^2}$
(d) ANSWER: $\frac{\partial z}{\partial x} = e^x y + 1 - \frac{3}{y}(-x^{-2})$
2. (a) HINT: Compute the slope of the diagonal line through the TR graph at $q = 6$.
ANSWER: approximately 1.75 dollars per item
(b) HINT: Compute the slope of the tangent line to the TR graph at $q = 6$ (i.e., compute the MR at $q = 6$).
ANSWER: approximately 1 dollar or 1 dollar per item
(c) HINT: The total revenue is always increasing, but the marginal revenue increases from $q = 0$ to $q = 3.5$ and then starts decreasing.
ANSWER: from $q = 3.5$ to $q = 7$
(d) HINT: Draw a reference line with slope 2 and find two quantities at which the tangent line to the TR graph is parallel to this reference line.
ANSWER: $q = 2$ and $q = 5$
(e) ANSWER: The graph of MR has q -intercepts at $q = 0$ and $q = 7$ and has a local maximum at the point $(3.5, 2.5)$.
3. (a) HINT: Compute $d(3 + h) - d(3)$.
ANSWER: $-\frac{1}{5}h^2 + \frac{4}{5}h$
(b) HINT: Divide the answer from part (a) by h .
ANSWER: $-\frac{1}{5}h + \frac{4}{5}$
(c) HINT: In your answer to part (b), let h go to 0.
ANSWER: $\frac{4}{5}$
4. (a) HINT: Compute $P'(q) = 4q^3 - 96q^2 + 540q$, set it equal to 0 and solve for q . Since there is a q in every term, you can factor out a q :
$$q(4q^2 - 96q + 540) = 0.$$
So, either $q = 0$ or $4q^2 - 96q + 540 = 0$. Use the quadratic formula to solve this second equation.
ANSWER: $q = 0, 9,$ and 15
(b) HINT: Evaluate the profit function at $q = 1, q = 9,$ and $q = 10$.
ANSWER: Min profit is \$39 at $q = 1$ thousand Lollipops; Max profit is \$4903 at $q = 9$ thousand Lollipops
(c) HINT: The Profit function is concave down when $P''(q) = 12q^2 - 192q + 540$ is negative. This quadratic is negative in between its roots. Use the quadratic formula to find where $P''(q)$ is equal to 0.
ANSWER: from $q = 3.64$ to $q = 12.36$
5. (a) HINT: Evaluate W at $p = 12400$ and $r = 0.062$.
ANSWER: \$821,004

(b) ANSWER: $W_p(p, r) = 100 - 45r - 0.005p$; $W_r(p, r) = -45p$

(c) HINT: Compute $W_p(15000, 0.054)$.

ANSWER: \$22.57

6. (a) ANSWER: $TR(q) = q^3 - 92q^2 + 2116q$; $MR(q) = 3q^2 - 184q + 2116$

(b) HINT: Use the quadratic formula to find where $3q^2 - 184q + 2116 = 0$. The only critical number of TR between 1 and 20 is $q = 15.333$. Plug this into $TR''(q) = 6q - 184$ to determine whether the critical number gives a local max or min.

ANSWER: $q = 15.333$ gives a local max

(c) HINT: $TC(q) = q^3 - 24q^2 + 192q + K$, for some constant K . $TC(28) = 8512 + K$ and $TR(28) = 9072$. Use the fact that $TC(28) = TR(28)$ to solve for K .

ANSWER: $FC = 560$ thousand dollars

7. (a) i. ANSWER: $\frac{8}{3}t^{3/2} - \frac{5}{3}t^3 + 27t^{1/3} + K$

ii. ANSWER: $\frac{3}{4}x^4 - \frac{8}{3}x^3 + \frac{9}{2}x^2 + K$

(b) HINT: Change in profit is given by the definite integral

$$\int_5^7 MR(q) - MC(q) dq.$$

ANSWER: 642 dollars

8. (a) ANSWER: $m = 9, 13$

(b) ANSWER: $F'(7) \approx 145$; $F''(1) \approx 85$

(c) ANSWER: $\int_6^8 f(x) dx \approx 260$

(d) $m = 3$