

Math 112 - Spring 2007  
Final Exam  
June 2, 2007

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Student ID Number: \_\_\_\_\_

1	15	
2	12	
3	12	
4	12	
5	12	
6	15	
7	12	
8	10	
Total	100	

- You are allowed to use a calculator, a ruler, and one 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method or read a value from a graph on your calculator when an algebraic method is available, you may not receive full credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 3 hours to complete the exam.

GOOD LUCK!

1. (15 pts) Find the following derivatives. Do not simplify your answers.

(a) Let  $A(r) = e^r \left( \frac{3}{r^2} + 5r^6 \right)$ . Find  $A'(r)$ .

ANSWER:  $A'(r) =$  \_\_\_\_\_

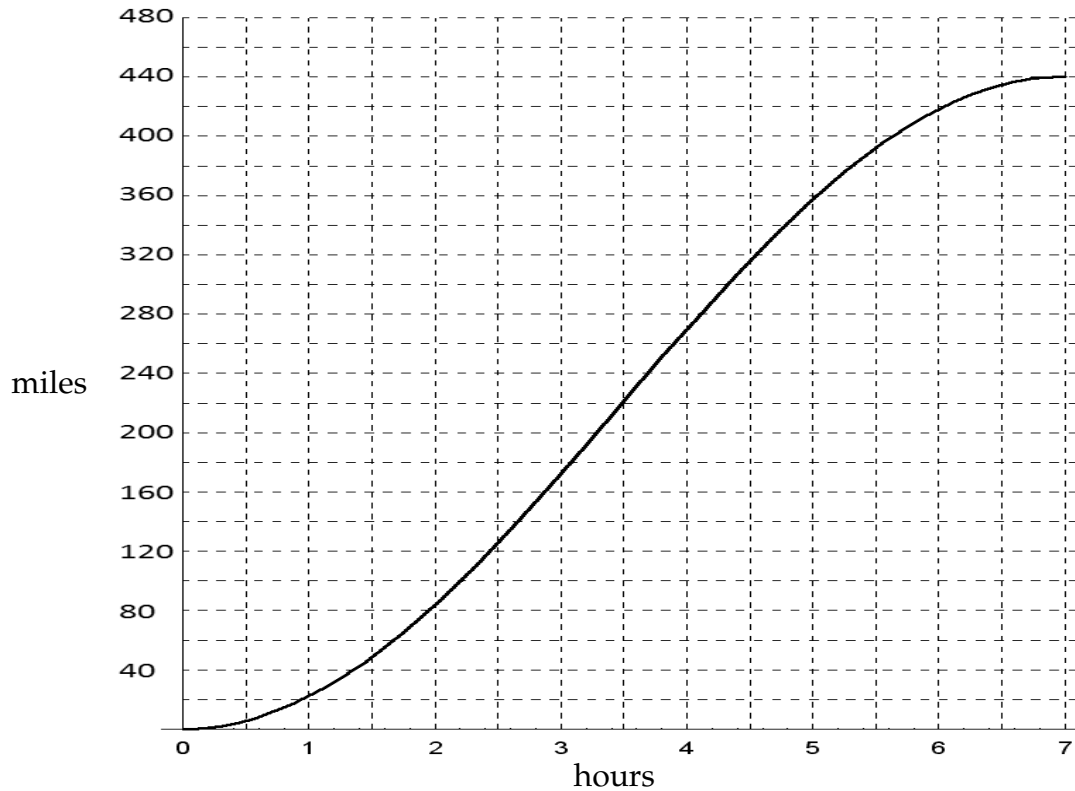
(b) Let  $f(t) = [\ln(3 - t^5)]^7$ . Find  $f'(t)$ .

ANSWER:  $f'(t) =$  \_\_\_\_\_

(c) Find the partial derivative  $\frac{\partial z}{\partial x}$  if  $z = \frac{x^2 y}{x + 1}$ .

ANSWER:  $\frac{\partial z}{\partial x} =$  \_\_\_\_\_

2. (12 pts) The graph below is the graph of the distance  $D$  (in miles) *vs.* time  $t$  (in hours) traveled by a truck over a 7 hour trip.



Use this graph and the methods studied in this class to answer the following questions. Be as precise as possible and indicate clearly how you get your answers. On the graphs provided, draw and label any lines or distances you use.

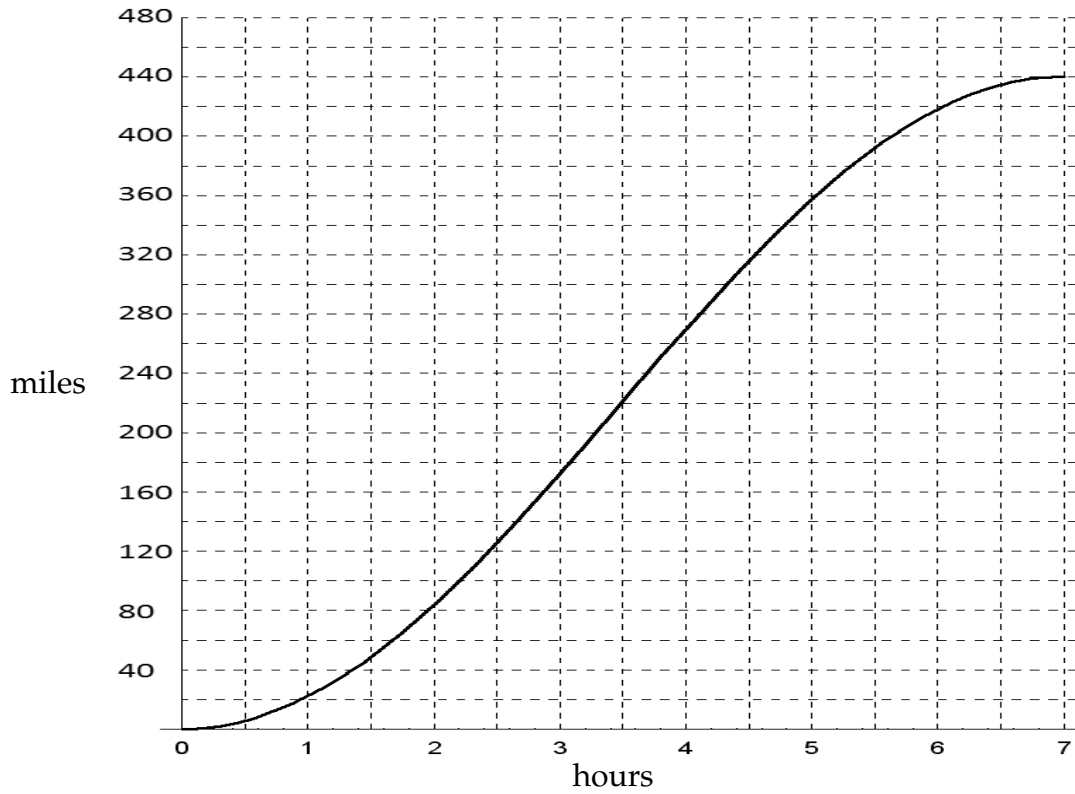
- (a) Estimate the car's Average Speed over the first 3 hours of the trip.

ANSWER: \_\_\_\_\_; UNITS: \_\_\_\_\_

- (b) Estimate the car's (instantaneous) speed at the time  $t = 3$  hours.

ANSWER: \_\_\_\_\_; UNITS: \_\_\_\_\_

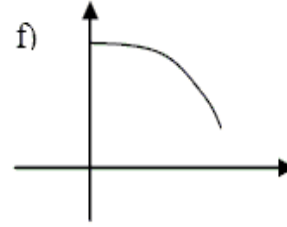
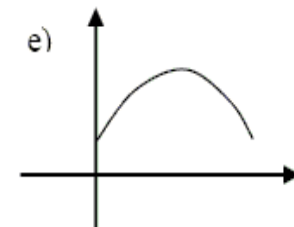
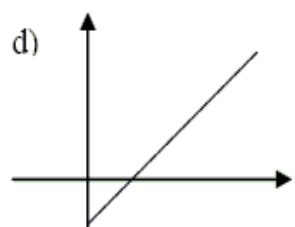
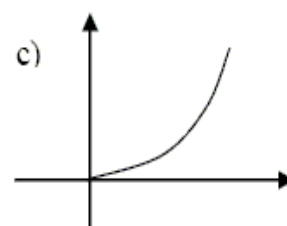
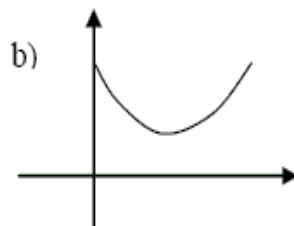
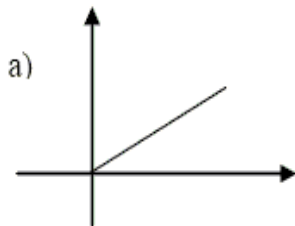
Here's the distance graph again:



(c) Find **all** times  $t$  when the car is driving at a speed of 60 miles per hour.

ANSWER: at  $t =$  \_\_\_\_\_ hours (*list all*)

(d) Which of the following graphs could be the graph of the (instantaneous) speed of this car? (no need to show work)



ANSWER: \_\_\_\_\_

3. (12 pts) The slope of the secant line for the function  $y = G(x)$  is given by

$$\frac{G(x+h) - G(x)}{h} = xh + 4x - 3h + 2.$$

(a) Find the value of  $\frac{G(4.02) - G(4)}{0.02}$ .

ANSWER:  $\frac{G(4.02) - G(4)}{0.02} = \underline{\hspace{2cm}}$

(b) Determine the value of  $G(8) - G(3)$ .

ANSWER:  $G(8) - G(3) = \underline{\hspace{2cm}}$

(c) Find a formula in terms of  $x$  for  $G'(x)$ .

ANSWER:  $G'(x) = \underline{\hspace{2cm}}$

(d) Determine the **slope of the tangent line** to  $G(x)$  at  $x = 5$ .

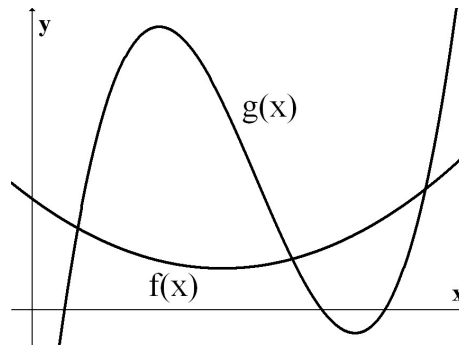
ANSWER: slope =  $\underline{\hspace{2cm}}$

4. (12 points)

To the right are rough sketches of the graphs of the two functions:

$$f(x) = 5x^2 - 50x + 200, \quad \text{and}$$

$$g(x) = 8x^3 - 142x^2 + 672x - 400.$$



(a) Find the longest interval where  $f(x)$  is increasing and  $g(x)$  is decreasing.

ANSWER: from  $x =$  \_\_\_\_\_ to  $x =$  \_\_\_\_\_

(b) Let  $P(x) = f(x) - g(x)$ . Is the graph of  $P(x)$  concave up, concave down, or neither at  $x = 5$ ? You must justify your answer to get credit.

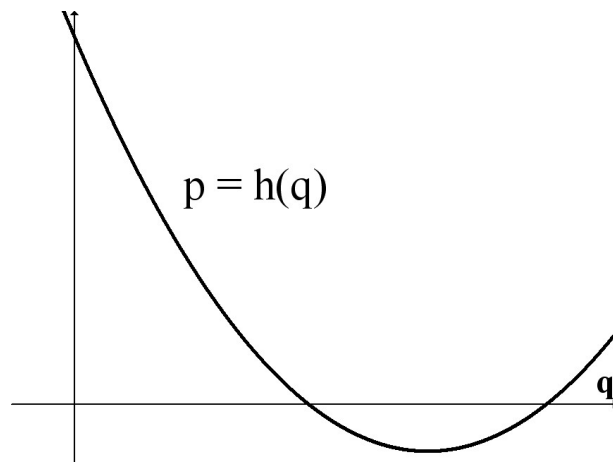
ANSWER (circle one):    CONCAVE UP        CONCAVE DOWN        NEITHER

5. (12 pts)

Your demand function for selling  $q$  Items is given by

$$p = h(q) = 2q^2 - 57q + 360,$$

where quantity  $q$  is measured in Items and price  $p$  is measured in dollars per Item. The graph of the demand function is given at right.



- (a) Find the largest interval with  $q \geq 0$  on which the demand function is positive and decreasing.

ANSWER: from  $q = 0$  to  $q =$  \_\_\_\_\_

- (b) Give the formula for the **Total Revenue** and **Marginal Revenue**.

ANSWER:  $TR(q) =$  \_\_\_\_\_

$MR(q) =$  \_\_\_\_\_

- (c) Find the **price** that corresponds to the quantity that gives the largest possible value of total revenue over the interval you obtained in part (a).

ANSWER: price = \_\_\_\_\_ dollars per Item

6. (15 points)

(a) Evaluate:  $\int \left( \frac{1}{x} - 5x^3 + 4\sqrt[3]{x} \right) dx$

ANSWER: \_\_\_\_\_

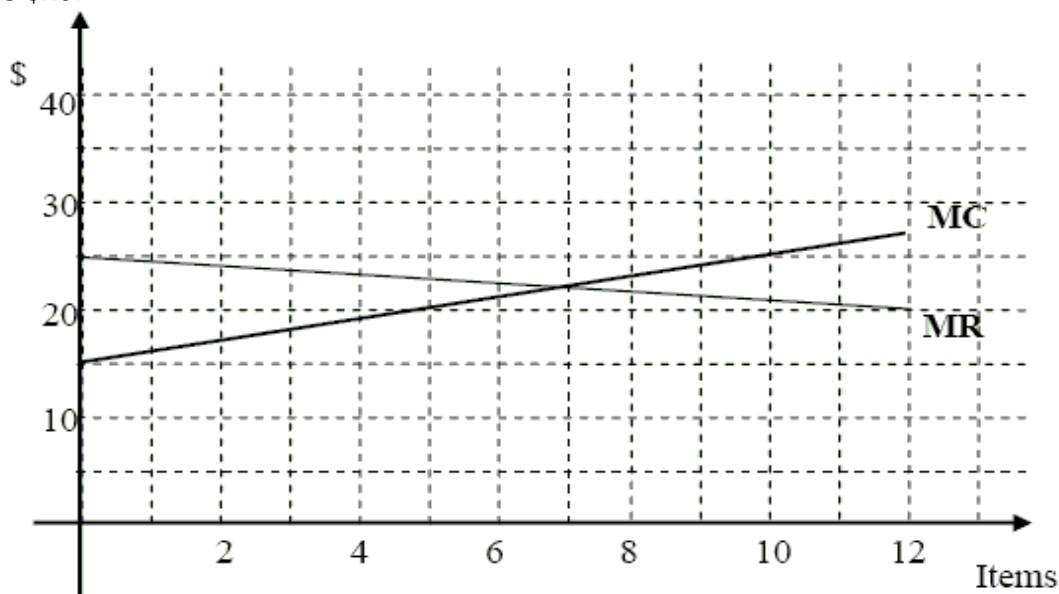
(b) Evaluate:  $\int_0^2 (3x^2 + 7x - 2) dx$

ANSWER: \_\_\_\_\_

(c) If the **marginal** cost function is  $MC(q) = \sqrt{q} + 5$  and the **fixed costs** are  $FC = 25$ , find the formula in terms of  $q$  for the **total cost**. (Solve for all constants of integration.)

ANSWER:  $TC(q) =$  \_\_\_\_\_

7. (12 pts) You produce and sell Hand Puppets in quantities of 1 to a dozen per day. The graphs below are your **Marginal Cost** and **Marginal Revenue** vs. quantity  $q$ . Your **fixed cost** is \$20.



- (a) Compute the **Total Cost** of producing 10 Puppets.

ANSWER:  $TC(10) =$  \_\_\_\_\_ dollars

- (b) Compute your **Profit** if you produce and sell 10 Puppets.

ANSWER:  $\text{Profit}(10) =$  \_\_\_\_\_ dollars

- (c) Which quantity from  $q = 0$  to  $q = 12$  Puppets will result in the largest **Total Revenue**?

ANSWER:  $q =$  \_\_\_\_\_ Puppets

- (d) Compute  $\int_0^{12} MR(q) dq$  and state what this value represents (in terms of cost and/or revenue).

ANSWER:  $\int_0^{12} MR(q) dq =$  \_\_\_\_\_

This value represents \_\_\_\_\_

8. (10 points) You have two final exams coming up, in your Calculus class and in your Sudoku class. You decide to hire tutors to help you study.

The math tutor charges \$15/hour, while the Sudoku tutor charges \$10/hour. For each hour you spend studying Calculus, your quarterly GPA (grade point average) increases by 0.2. For each hour you study Sudoku, your GPA increases by 0.1 points. You have a total of at most 20 hrs that you can dedicate to your studying, and a budget of at most \$250.

Let  $x$  denote the number of hours you spend studying Calculus, and let  $y$  denote the number of hours you spend studying Sudoku. You want to maximize the increase in your GPA,  $G(x, y)$ , subject to the given constraints.

- (a) Find the formula for your objective function  $G(x, y)$ .

$$G(x, y) =$$

- (b) Write the inequalities for your constraints.

- (c) Sketch the feasible region and label the coordinates of the vertices.

- (d) Find the maximum increase in your GPA and how many hours you should spend studying for each midterm to achieve this maximum GPA improvement.

ANSWER:  $x =$  \_\_\_\_\_ hrs for Calculus  
 $y =$  \_\_\_\_\_ hrs for Sudoku  
GPA increase = \_\_\_\_\_ points