

MATH 112 – FINAL EXAM Hints and Answers  
Spring 2007

1. (a) ANSWER:  $A'(r) = e^r \left( \frac{3}{r^2} + 5r^6 \right) + e^r (-6r^{-3} + 30r^5)$   
(b) ANSWER:  $f'(t) = 7[\ln(3 - t^5)]^6 \left( \frac{1}{3-t^5} \right) (-5t^4)$   
(c) ANSWER:  $\frac{\partial z}{\partial x} = \frac{2xy(x+1) - x^2y}{(x+1)^2}$
2. (a) HINT: Compute the slope of the secant line through the distance graph at  $t = 0$  and  $t = 3$ .  
ANSWER: approximately 57.33 miles per hour  
(b) HINT: Compute the slope of the line tangent to the distance graph at  $t = 3$ .  
ANSWER: approximately 96 miles per hour  
(c) HINT: Draw a reference line with slope 60 and find all values of  $t$  at which the line tangent to the distance graph is parallel to your reference line.  
ANSWER: (approximate)  $t = 1.5$  and  $5.5$  hours  
(d) ANSWER:  $e$
3. (a) HINT: Plug  $x = 4$  and  $h = 0.02$  into the formula for  $\frac{G(x+h) - G(x)}{h}$ .  
ANSWER: 18.02  
(b) HINT:  $G(x+h) - G(x) = (xh + 4x - 3h + 2)h$ . Plug  $x = 3$  and  $h = 5$  into this formula for  $G(x+h) - G(x)$ .  
ANSWER: 70  
(c) HINT: Let  $h$  go to 0 in the formula for  $\frac{G(x+h) - G(x)}{h}$ .  
ANSWER:  $G'(x) = 4x + 2$   
(d) HINT: Plug  $x = 5$  into the formula for  $G'(x)$ .  
ANSWER: 22
4. (a) HINT:  $f(x)$  is increasing for  $x$  larger than 5.  $g(x)$  is decreasing from  $x = 3.27$  to  $x = 8.56$ .  
ANSWER: from  $x = 5$  to  $x = 8.56$   
(b) HINT:  $P(x) = -8x^3 + 147x^2 - 722x + 600$  and  $P''(x) = -48x + 294$ . Plug  $x = 5$  into the formula for  $P''(x)$  and determine its sign.  
ANSWER: concave up
5. (a) ANSWER: from  $q = 0$  to  $q = 9.447$   
(b) ANSWER:  $TR(q) = 2q^3 - 57q^2 + 360q$ ;  $MR(q) = 6q^2 - 114q + 360$   
(c) HINT: Set  $MR(q) = 0$  and solve for  $q$ . You should find that  $MR(q) = 0$  at  $q = 4$  and  $q = 15$ . We are only concerned about values of  $q$  between 0 and 9.447, so we can ignore  $q = 15$ . Plug the endpoints  $q = 0$  and  $q = 9.447$  and the critical number  $q = 4$  into the formula for  $TR(q)$  to find the quantity that gives maximum total revenue ( $q = 4$ ). Then plug  $q = 4$  into the demand function  $h(q)$ .  
ANSWER: 164 dollars per item
6. (a) ANSWER:  $\ln x - \frac{5}{4}x^4 + 3x^{4/3} + K$   
(b) ANSWER: 18

- (c) ANSWER:  $TC(q) = \frac{2}{3}q^{3/2} + 5q + 25$
7. (a) HINT: Compute the area below  $MC$  from  $q = 0$  and  $q = 10$  and add the fixed costs.  
ANSWER:  $TC(10) = 220$  dollars
- (b) HINT: Profit at  $q = 10$  is  $TR(10) - TC(10)$ . To compute  $TR(10)$ , compute the area under  $MR$  from  $q = 0$  to  $q = 10$ . You computed  $TC(10)$  in part (a).  
ANSWER: Profit(10) = 10 dollars
- (c) HINT: Since  $MR$  is always positive,  $TR$  is always increasing. So,  $TR$  will be largest at the right-hand endpoint of the interval.  
ANSWER:  $q = 12$  Puppets
- (d) ANSWER:  $\int_0^{12} MR(q) dq = 270$ . This represents  $TR(12)$ .
8. (a) ANSWER:  $G(x, y) = 0.2x + 0.1y$
- (b) ANSWER:  $x + y \leq 20$  and  $15x + 10y \leq 250$
- (c) ANSWER: The vertices are  $(0, 20)$ ,  $(10, 10)$ , and  $(16.67, 0)$ .
- (d) ANSWER:  $x = 16.67$  hours for Calculus,  $y = 0$  hours for Sudoku, GPA increases 3.33 points