MATH 112 – FINAL EXAM Hints and Answers
Spring 2008

1. (a) ANSWER: $A(t) = 0.16t^3 - 3.6t^2 + 20.25t + K$

   (b) ANSWER: $A(0) = 397.5$

   (c) HINT: Set $R(t) = 0$ and solve for t to find the critical numbers. Then apply the Second Derivative Test to see whether each critical number gives a local max or min.

   ANSWER: $t = 3.75$ gives a local max; $t = 11.25$ gives a local min

   (d) HINT: Sketch a graph of $R(t)$, a parabola that opens up and intersects the t-axis at $t = 3.75$ and 11.25. The balloon is falling most rapidly at the vertex of this parabola.

   ANSWER: $t = 7.5$ hours

   (e) HINT: Since $R(t)$ is negative on the interval from $t = 4$ to $t = 6$, the balloon is falling during this time. To determine how far it falls, compute $A(6) - A(4)$, the change in altitude from $t = 4$ to $t = 6$.

   ANSWER: The balloon falls 7.18 feet.

2. (a) HINT: $VC(4) - VC(2) = \displaystyle\int_2^4 MC(q)\,dq$

   ANSWER: 24 Hundred Dollars

   (b) HINT: $TC(q)$ is an anti-derivative of $MC(q)$. So, $TC(q) = q^3 - 11q^2 + 50q + K$, for some value of K. Use the fact that $TC(5.5) = 158.625$ to find the correct value of K and then $FC = TC(0)$.

   ANSWER: $FC = 50$ Hundred Dollars

   (c) HINT: Find the q at which $MR(q) = 0$.

   ANSWER: $q = 10$ Hundred Things

   (d) HINT: Set $MR(q) = MC(q)$ to find the value of q that maximizes profit: $q = 8$. Then, max profit is

   $$\int_0^8 MR(q)\,dq - \int_0^8 MC(q)\,dq - FC.$$

   ANSWER: 622 Hundred Dollars

3. HINT: Let x be the amount of Vanilla Bean you produce (in gallons) and y the amount of Double Chocolate. Your constraints are $10x + 8y \leq 3380$ and $12x + 15y \leq 5190$ and your objective function is $P(x, y) = \$2.25x + 1.75y$. The vertices of your feasible region are $(0, 0)$, $(0, 346)$, $(170, 210)$, and $(338, 0)$. Profit is maximized at $(338, 0)$.

   ANSWER: \$760.50

4. (a) HINT: Set $g'(x) = 0$ and solve for x.

   ANSWER: $x = 4$ and 8

   (b) HINT: Sketch rough graphs of $f'(x)$ and $g'(x)$. The graph of $f'(x)$ is a line with positive slope and positive y-intercept. So, $f'(x)$ is always positive and, thus, $f(x)$ is always increasing. The graph of $g'(x)$ is a parabola that opens upward and intersects the x-axis at $x = 4$ and $x = 8$ (as you found in part (a)). So, $g'(x)$ is negative, and thus $g(x)$ is decreasing, from $x = 4$ to $x = 8$.

   ANSWER: from $x = 4$ to $x = 8$

   (c) HINT: $f'(x)$ is positive from $x = 0$ to $x = 10$. So, $f(x)$ is increasing on this interval.

   ANSWER: $x = 0$

(d) ANSWER: $5b^2 + 66.25b$

(e) HINT: Concavity is given by the second derivative. $g''(x) = 10x - 60$. We need to know where $g''(x)$ is negative. Sketch a rough graph of $g''(x)$: it's a line with positive slope and negative y-intercept. So, $g''(x)$ is negative from $x = 0$ to its x-intercept.
ANSWER: from $x = 0$ to $x = 6$

5. (a) ANSWER: $16\sqrt{t} + 9t^{2/3} + \frac{9}{5}\ln t + K$

(b) ANSWER: 51

(c) ANSWER: 0.9167

6. (a) ANSWER: $\dfrac{dy}{dx} = \dfrac{x^5\left[4xe^x + e^x \cdot 4 - \frac{2x}{x^2+3}\right] - \left[4xe^x - \ln(x^2+3)\right](5x^4)}{x^{10}}$

(b) ANSWER: $\dfrac{\partial z}{\partial y} = x + \dfrac{9e^y}{x} + \dfrac{8e^x}{y^2}$

(c) ANSWER: $f'(m) = \dfrac{-13}{(7+m)^2}$