

MATH 112
Final Exam
June 6, 2009

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	12	
2	12	
3	19	
4	12	
5	15	
6	12	
7	18	
Total	100	

- Your exam should consist of this cover sheet, followed by seven problems on nine pages. Check that you have a complete exam.
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. Clearly label lines and points that you are using, shade areas, and show all calculations. The correct answer with no supporting work may result in no credit.
- If you compute an area by counting squares or adding the areas of several trapezoids, explicitly state your method and clearly show all relevant calculations.
- If you use a guess-and-check method or read a value from a graph on your calculator when an algebraic method is available, you may not receive full credit.

GOOD LUCK!

1. (12 points) Evaluate each integral.

(a) $\int \left(\frac{4}{x^5} - \frac{5}{\sqrt[3]{x}} + 2e^x \right) dx$

(b) $\int_{-1}^1 (10x^5 - 4x^3) dx$

(c) $\int_{e^2}^{e^5} \frac{1}{3x} dx$

2. (12 points) Compute the indicated derivative. DO NOT SIMPLIFY.

(a) $y = \frac{\ln(x^2 - 4x)}{(x^5)(\sqrt{3x + 1})}$

$$\frac{dy}{dx} =$$

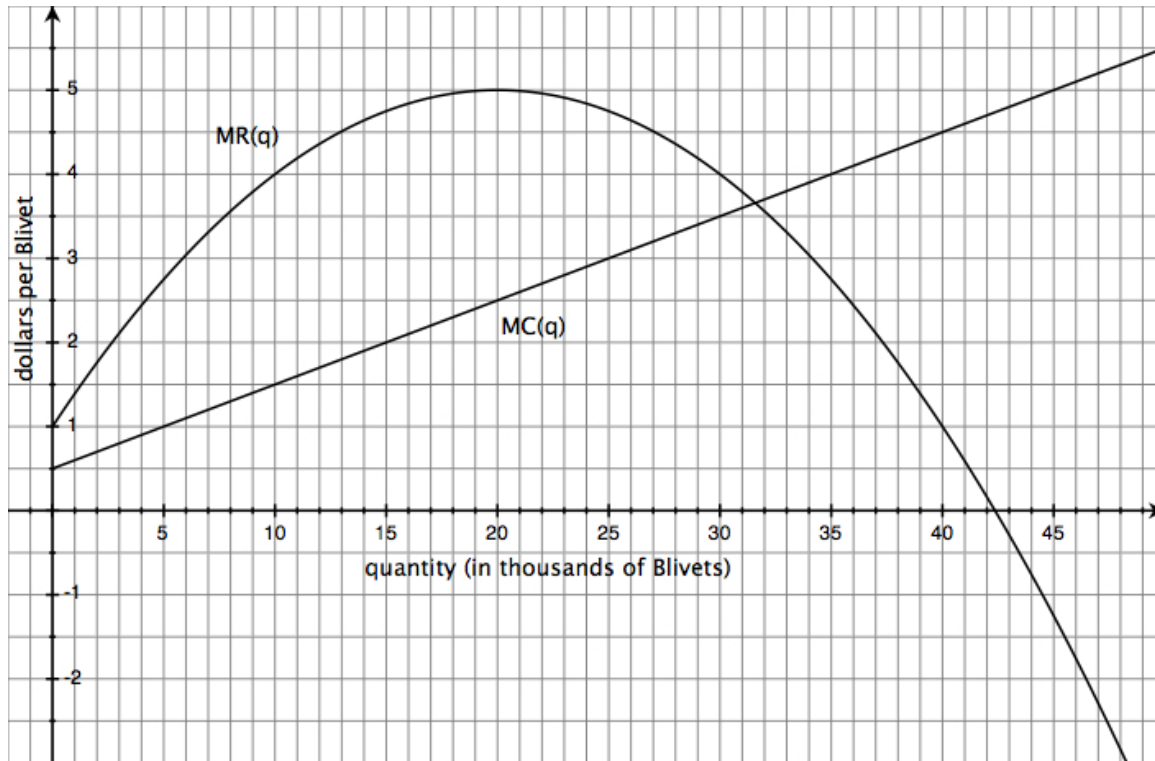
(b) $h(v) = \frac{v^2 + 3v}{v^3 - 5v} + ve^{2v}$

$$h'(v) =$$

(c) $A(m) = \int_0^m e^x - \frac{5}{\sqrt{x}} dx$

$$A'(m) =$$

3. (19 points) Below are the graphs of marginal revenue and marginal cost for producing and selling Blivets.



- (a) Give all quantities at which each of the following functions has a horizontal tangent line. (“none” is a possible answer.)
- $TR(q)$: $q =$ _____
 - $TC(q)$: $q =$ _____
 - $MR(q)$: $q =$ _____
 - profit $P(q)$: $q =$ _____
- (b) Compute the total revenue you earn selling five thousand Blivets.

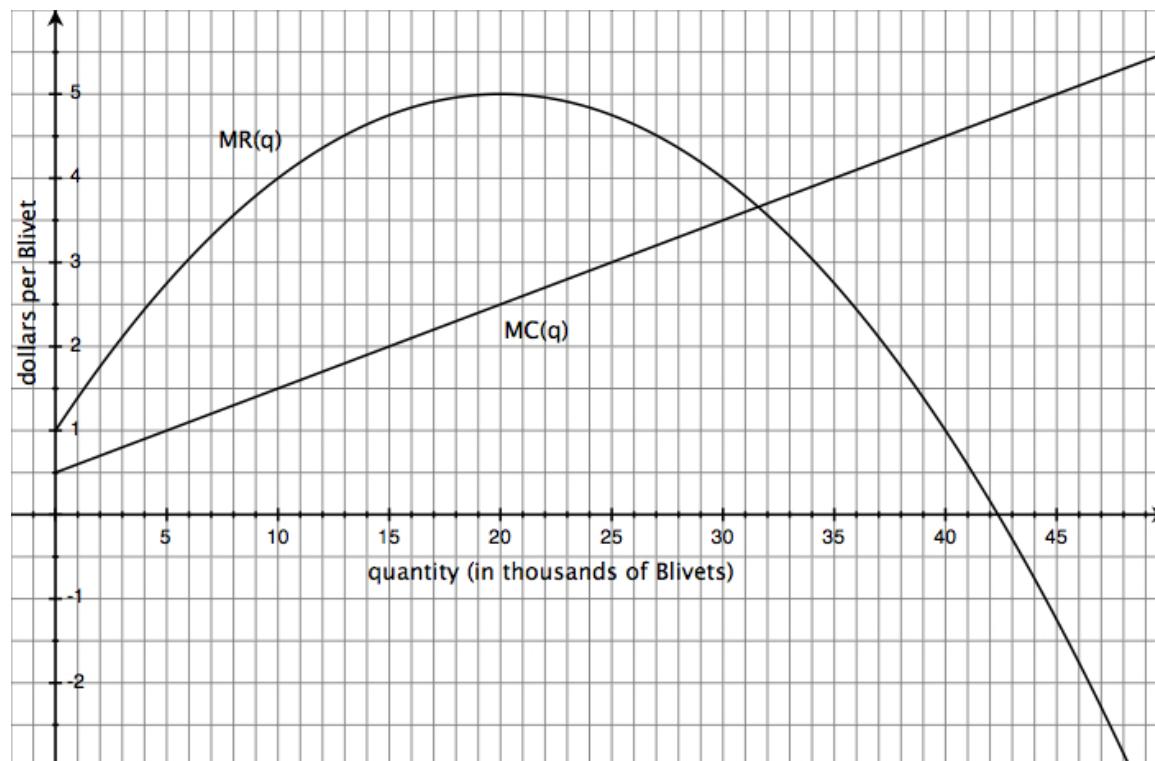
ANSWER: _____ thousand dollars

- (c) How much does your profit increase if quantity changes from 25 to 30 thousand Blivets?

ANSWER: _____ thousand dollars

THIS PROBLEM CONTINUES ON THE NEXT PAGE.

Here are the graphs of MR and MC again:



(d) Your fixed costs are 2.5 thousand dollars. Find formulas for $MC(q)$, $VC(q)$ and $TC(q)$.

ANSWERS: $MC(q) =$ _____

$VC(q) =$ _____

$TC(q) =$ _____

(e) Recall that the formula for average cost is given by

$$AC(q) = \frac{TC(q)}{q}.$$

Find the quantity at which average cost has a local minimum.

ANSWER: $q =$ _____ thousand Blivets

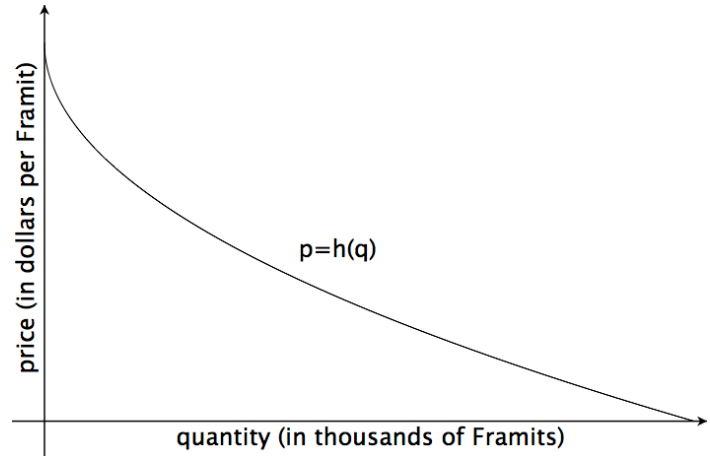
4. (12 points)

The Demand Curve for selling Framits (shown at right) has the formula

$$p = h(q) = 20 - 2\sqrt{q},$$

where q is in thousands of Framits and p is in dollars per Framit. The total cost (in thousands of dollars) to produce q thousand Framits is given by the formula

$$TC(q) = 2q + 5.$$



- (a) We can see from the graph that the price per Framit is always decreasing. Find the longest interval, beginning at $q = 0$, on which the price per Framit is also positive.

ANSWER: from $q = 0$ to $q =$ _____

- (b) Find the quantity at which total revenue is largest.

ANSWER: $q =$ _____ thousand Framits

- (c) Let $P(q)$ denote the profit (in thousands of dollars) you earn by selling q thousand Framits. Find all critical numbers of profit and use the Second Derivative Test to determine whether each critical number gives a local maximum or a local minimum of $P(q)$.

5. (15 points) Suppose $f(x, y) = 8x^3 - 9x^2 + 3y^2 + 12xy + 10$.

(a) Write out the partial derivatives of $f(x, y)$.

ANSWER: $f_x(x, y) =$ _____

$f_y(x, y) =$ _____

(b) Use a partial derivative to approximate the value of

$$\frac{f(2.0003, 2) - f(2, 2)}{0.0003}.$$

ANSWER: _____

(c) Which function is steeper?

i. $h(t) = f(3, t)$ at $t = 0$; or

ii. $k(t) = f(t, 4)$ at $t = 1$.

ANSWER: (circle one) i ii

(d) Find all candidates (x, y) that could give a local maximum or a local minimum of $f(x, y)$.

ANSWER: _____ (list all pairs)

6. (12 points) The distance formulas for two rocket cars are given by:

$$E(t) = 30t - 1.5t^2 \text{ and } F(t) = 2t + 2t^2,$$

where t is measured in minutes and distance is measured in feet.

(a) Write out the formula for the average speed of the E -car over the h -minute interval beginning at $t = 2$:

$$\frac{E(2+h) - E(2)}{h}.$$

Simplify your formula as much as possible.

ANSWER: $\frac{E(2+h) - E(2)}{h} =$ _____

(b) Find the time at which the F car's instantaneous speed exceeds the E car's instantaneous speed by exactly 2.5 feet per minute.

ANSWER: $t =$ _____ minutes

7. (18 points)

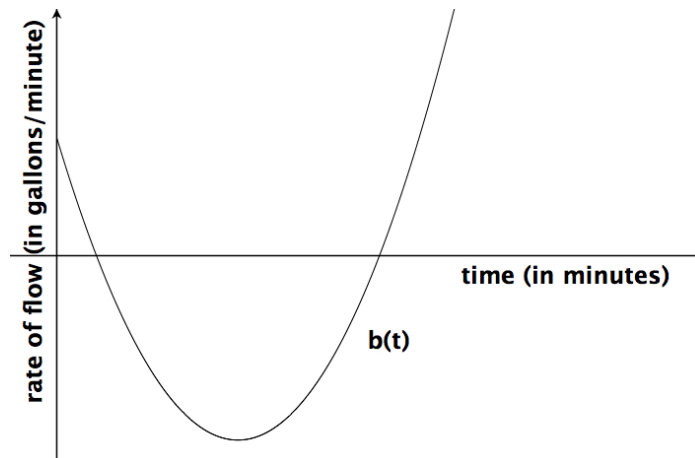
Water flows into and out of two vats: vat A and vat B . The **amount of water** in vat A is given by the formula

$$A(t) = -2.5t^2 + 15t + 75,$$

where t is in minutes and the amount is measured in gallons. The **rate of flow** for vat B is given by the formula

$$b(t) = 20t^2 - 73t + 26,$$

where t is in minutes and the rate of flow is in gallons per minute. The graph of vat B 's rate of flow is shown at right.



- (a) Find all times at which the **amount of water** in vat B reaches a local maximum or minimum value

ANSWER: $t =$ _____ minutes (list all)

- (b) At what time is the water level in both vats **dropping at the same rate**?

ANSWER: $t =$ _____ minutes

THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.

Here are those formulas again:

- amount of water in vat A : $A(t) = -2.5t^2 + 15t + 75$
- rate of flow in and out of vat B : $b(t) = 20t^2 - 73t + 26$

(c) Explain what $\int_4^5 b(t) dt$ represents in terms of the amount of water in vat B .

(d) At $t = 0$ the vats contain the same amount of water. How much water is in vat B at $t = 2$ minutes?

ANSWER: _____gallons

(e) What is the maximum water level in vat A ?

ANSWER: _____gallons