MATH 112 – FINAL EXAM Hints and Answers
Version Alpha
Spring 2009

1. (a) ANSWER: $-\dfrac{1}{x^4} - \dfrac{15}{2}x^{2/3} + 2e^x + K$
   (b) ANSWER: 0
   (c) ANSWER: 1

2. (a) ANSWER: $\dfrac{dy}{dx} = \dfrac{(x^5)(\sqrt{3x+1})\frac{2x-4}{x^2-4x} - \ln(x^2-4x)\left[(x^5)\frac{1}{2}(3x+1)^{-1/2}(3) + \sqrt{3x+1}(5x^4)\right]}{\left[(x^5)\sqrt{3x+1}\right]^2}$
   (b) ANSWER: $h'(v) = \dfrac{(v^3-5v)(2v+3) - (v^2+3v)(3v^2-5)}{(v^3-5v)^2} + (v)(e^{2v})(2) + (e^{2v})(1)$
   (c) ANSWER: $A'(m) = e^m - \frac{5}{\sqrt{m}}$

3. (a) ANSWERS:
   - $TR(q)$: $q = 42.5$
   - $TC(q)$: none
   - $MR(q)$: $q = 20$
   - profit $P(q)$: $q = 31.5$

   (b) HINT: Compute the area under the $MR$ graph from $q = 0$ to $q = 5$.
   ANSWER: approximately 9.375 thousand dollars
   (c) HINT: Compute the area between the $MR$ and $MC$ graphs from $q = 25$ to $q = 30$.
   ANSWER: approximately 5.625 thousand dollars
   (d) ANSWERS: $MC(q) = 0.1q + 0.5$, $VC(q) = 0.05q^2 + 0.5q$, $TC(q) = 0.05q^2 + 0.5q + 2.5$
   (e) HINT: $AC(q) = 0.05q + 0.5 + \frac{2.5}{q}$. Compute $AC'(q)$, set it equal to 0, and solve for $q$ to find the critical number of $AC(q)$. Use the second derivative test to show that this gives a local min of $AC$.
   ANSWER: $q = 7.07$ thousand Blivets

4. (a) HINT: The price function is positive from $q = 0$ until its graph hits the $q$-axis. This happens where the price is equal to 0. So, set $h(q) = 0$ and solve for $q$.
   ANSWER: from $q = 0$ to $q = 100$
   (b) HINT: $TR(q) = h(q) \cdot q = (20 - 2\sqrt{q})q = 20q - 2q^{3/2}$. Compute $TR'(q)$, set it equal to 0, and solve for $q$ to get the critical number of $TR$. It is clear from the demand curve (and our interpretation of $TR$ as a certain rectangle) that this critical number will give a maximum. (You could also use the Second Derivative Test to verify that this critical number gives a local max of $TR$.)
   ANSWER: $q = 44.444$ thousand Framits
   (c) HINT: $P(q) = (20q - 2q^{3/2}) - (2q + 5) = 18q - 2q^{3/2} - 5$.
   ANSWER: $q = 36$ gives a local maximum of profit

5. (a) ANSWER: $f_x(x, y) = 24x^2 - 18x + 12y$, $f_y(x, y) = 6y + 12x$
   (b) HINT: $\dfrac{f(2.0003, 2) - f(2, 2)}{0.0003} \approx f_x(2, 2)$
   ANSWER: 84
   (c) HINT: You need to investigate the slopes of tangent lines.

i. $h'(t) = f_y(3,t)$. So, $h'(0) = f_y(3,0) = 36$

ii. $k'(t) = f_x(t,4)$. So, $k'(1) = f_x(1,4) = 54$

ANSWER: ii is steeper

(d) HINT: Set $f_x$ and $f_y$ equal to 0 and solve the resulting system of equations.
ANSWER: $(0,0)$ and $(1.75, -3.5)$

6. (a) ANSWER: $24 - 1.5h$

   (b) HINT: Solve for $t$: $F'(t) - E'(t) = 2.5$.
   ANSWER: $t = 4.36$ minutes

7. (a) HINT: Find the times at which $b(t) = 0$.
   ANSWER: $t = 0.4$ and $3.25$ minutes

   (b) HINT: The rate of flow for vat $A$ is: $a(t) = -5t + 15$. Set $a(t)$ equal to $b(t)$ and solve for $t$. You should get two times. At one of these times, the water levels in the vats are rising (because their rates of flow are positive). At the other, the levels are dropping (because their rates of flow are negative).
   ANSWER: $t = 3.23$ minutes

   (c) ANSWER: $\int_4^5 b(t)\,dt$ represents the **change in the amount** of water in Vat $B$ from $t = 4$ to $t = 5$.

   (d) HINT: $B(t) = \frac{20}{3}t^3 - \frac{73}{2}t^2 + 26t + K$, for some constant $K$. Since $A(0) = 75$ and both vats contain the same amount of water at $t = 0$, $B(0)$ is also equal to 75. Use this fact to find $K$ and then compute the value of $B(2)$.
   ANSWER: $B(2) = 34.333$ gallons

   (e) Find the "$y$"-coordinate of the vertex of $A(t)$.
   ANSWER: 97.5 gallons