

NAME: _____

QUIZ SECTION: _____

Student ID #: _____

**Math 112 -- Spring 2011
Final Exam**

HONOR STATEMENT:

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

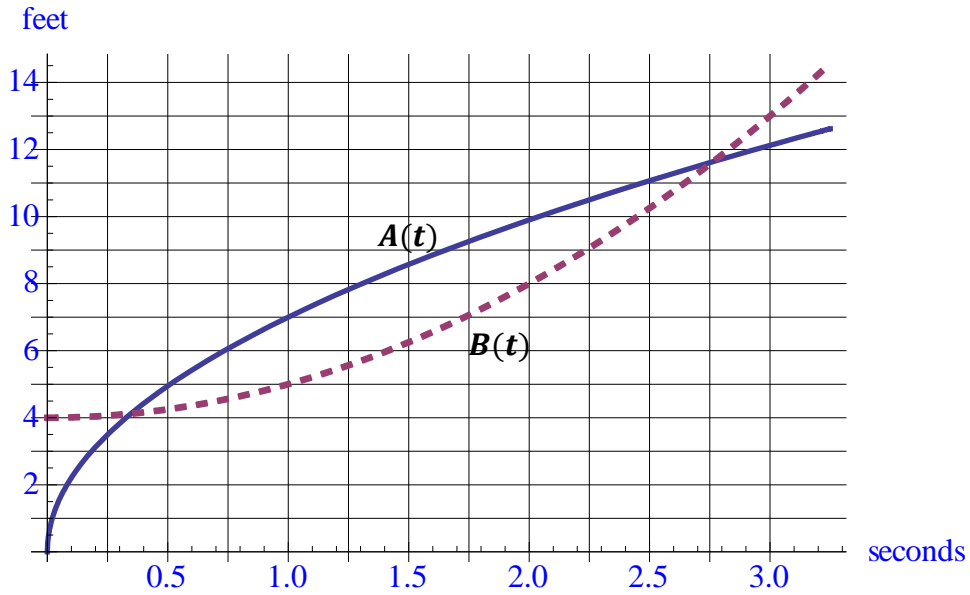
INSTRUCTIONS:

- When the exam starts, **verify that your exam contains 8 pages** (including this cover page).
- Please **turn your cell phone OFF** and put it away for the duration of the exam.
- Unless otherwise indicated, you **must show all your work in order to get full credit**. The correct answer with incorrect or missing work may result in little or no credit.
- On problems in which you use a graph, show your work by clearly drawing & labeling any lines and points you use.
- If you use a graphing calculator or a guess-and-check method when an algebraic method is available, you will not receive full credit.
- Unless otherwise specified, **you may round your final answer to two decimal digits**.
- You are allowed to use a calculator, a ruler, and one sheet of notes. You have up to 3 hours for this exam.

GOOD LUCK!

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Total:	100	

1) (14 pts) The following are the **graphs of the distance** (in feet) vs. time (in seconds) for two cars, *A* and *B*.



Use these graphs and the methods studied in this class to answer the following questions. Draw and label on the graphs any lines you use, and be as precise as possible.

a) (8 pts) Estimate the following:

i. the average speed of **car B** from 1.5 to 2 seconds is: _____ feet/sec

ii. the value of $\frac{B(1.001) - B(1)}{0.001}$ is: _____

iii. the time *t* when the speed of **car A** is 8 ft/sec is: _____ seconds

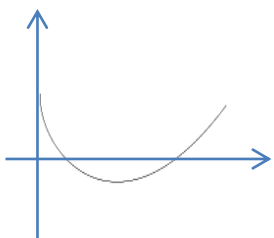
iv. the longest time interval over which car A is ahead of car B and car A drives faster than car B is: from _____ to _____ seconds

b) (6 pts) Consider the four rough graphs below, labeled (a)-(d). No need to show any work.

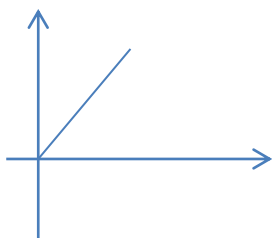
Which graph could be the **derived graph of $A(t)$** : (a), (b), (c), (d), or **NONE** of them: _____

Which graph could be the **derived graph of $B(t)$** : (a), (b), (c), (d), or **NONE** of them: _____

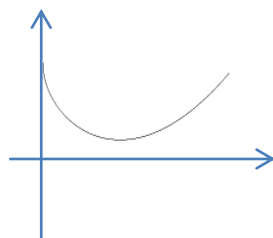
Which graph could be the **graph of $A(t) - B(t)$** : (a), (b), (c), (d), or **NONE** of them: _____



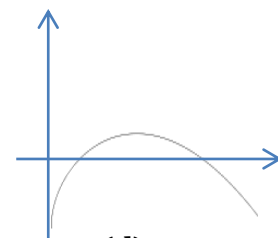
(a)



(b)



(c)



(d)

2) (12 pts)

a) Suppose $f(x) = -x^2 + 2x - 7$.

- i. (4 pts) Compute the formula in terms of x and h for $\frac{f(x+h)-f(x)}{h}$.
Show work and simplify as far as possible.

Answer: $\frac{f(x+h)-f(x)}{h} =$ _____

- ii. (2 pts) Explain how you can use your answer from part (a) to obtain a formula for $f'(x)$.

b) You do not know the formula for another function, $g(x)$, but you do know that the formula for the slope of the secant line to $g(x)$ from $x = m$ to $x = m + h$ is given by

$$\frac{g(m+h) - g(m)}{h} = \frac{6}{(m+2)(m+h+1)}$$

- i. (3 pts) Find a formula in terms of k for $g(2+k) - g(2)$.

Answer: $g(2+k) - g(2) =$ _____

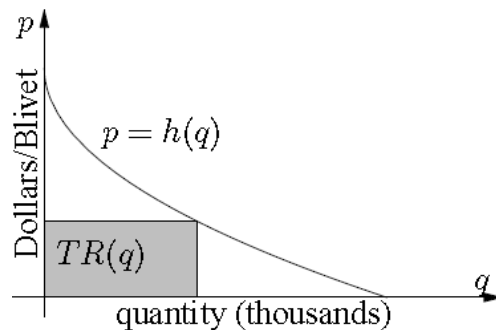
- ii. (3 pts) Compute the slope of the tangent line to $g(x)$ at $x = 5$.

Answer: slope = _____

3) (14 pts) The demand curve for selling Blivets has the formula

$$p = h(q) = 24 - 4\sqrt{q},$$

where q is measured in thousands of Blivets, and the price per item p is measured in dollars. Each Blivet costs \$6 to produce, and your fixed costs are $FC = 1$ thousand dollars.



- a) (3pts) Find the interval on which the demand curve makes sense (i.e. the demand curve is greater or equal to zero and decreasing.)

Answer: From $q =$ _____ to $q =$ _____ thousand blivets

- b) (3 pts) Write formulas for the total revenue $TR(q)$, and for $MR(q)$. Simplify your answers and include correct units. No need to show work.

$$TR(q) = \text{_____ measured in _____}$$

$$MR(q) \cong \text{_____ measured in _____}$$

- c) (4 pts) What quantity q results in the maximum total revenue? Justify your answer.

Answer: $q =$ _____ thousand Blivets

- d) (4 pts) Determine the maximum profit possible.

Answer: Max profit is _____ thousand dollars

4) (16 points)

a) Let $f(x) = \frac{x^2 - e^x}{\sqrt{x^3 + \ln(x)}}$. Compute $f'(x)$. Do not simplify, but box your final answer.

b) Let $g(t) = 2t\sqrt{t^2 + 5}$. Compute $g'(2)$. Show your work.

Answer: $g'(2) =$ _____

c) Let $z = \frac{3x}{y} - x^2y^2 + 2 \ln y$. Compute and simplify $\frac{\partial z}{\partial x}$.

Answer: $\frac{\partial z}{\partial x} =$ _____

d) A certain multi-variable function $y = f(r, s)$ has these two partial derivatives:

$$\frac{\partial y}{\partial r} = \frac{1}{r+1} + \ln(s), \text{ and } \frac{\partial y}{\partial s} = \frac{r}{s} + 2s$$

Compute the slope of the tangent line to the graph of $y = f(r, 3)$ at $r = 1$.

Answer: _____

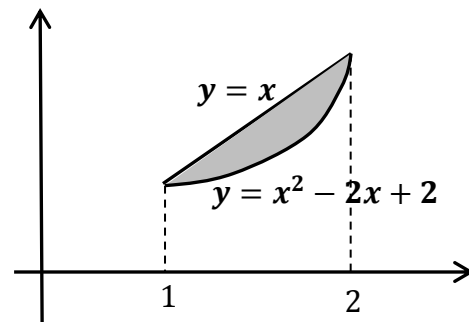
5) (12 pts)

a) Compute each of the following **integrals**. Simplify and box your final answers.

i. $\int \left(\sqrt{2} t - \frac{5}{t} + 2\sqrt{t} \right) dt$

ii. $\int_0^2 ((x+1)(x^2-5)) dx$

b) Compute the shaded area.
Show work and box your final answer.



6) (16 pts) A balloon moves up and down. Its **rate-of-ascent** (speed) at time t hours is given by the function

$$a(t) = \frac{t^2}{2} - 10t + 35 \text{ (in feet/hour)}$$

Let $A(t)$ denote the altitude of this balloon at t hours.

a) Compute the change in the altitude of this balloon from 1 to 2 hours, $A(2) - A(1)$.

ANSWER: $A(2) - A(1) =$ _____ feet

b) Suppose $A(6) = 442$ feet. Compute the initial altitude of this balloon, $A(0)$.

ANSWER: $A(0) =$ _____ feet

c) Find the candidates for the local minimum and the local maximum of the altitude $A(t)$ of the balloon. For each, use the second derivative test to determine if they are a local minimum or a local maximum.

ANSWER: $A(t)$ has a local minimum at $t =$ _____ hours

local maximum at $t =$ _____ hours

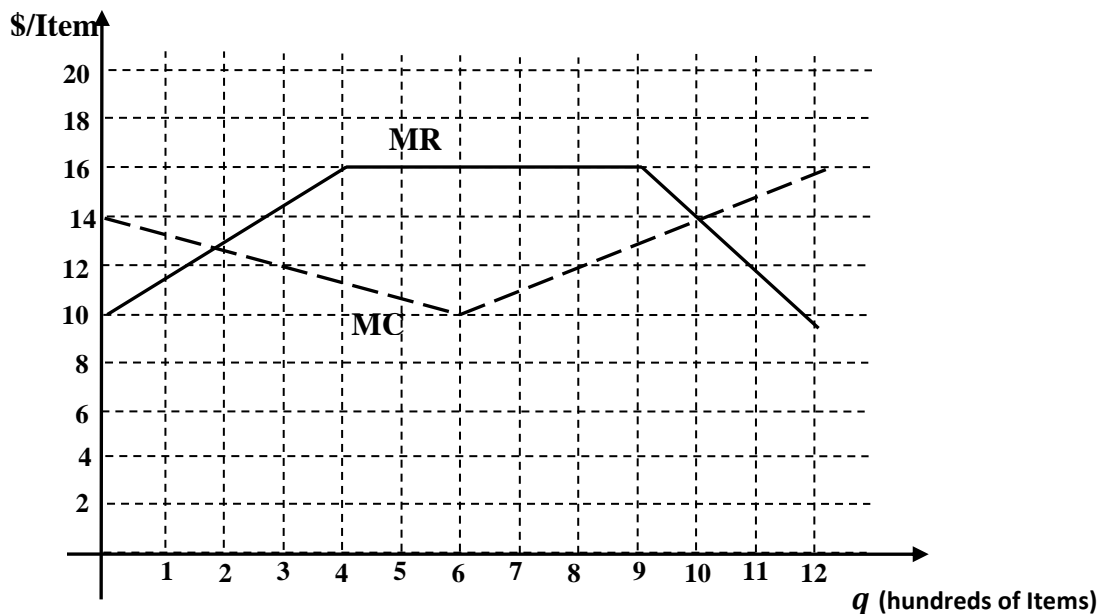
d) Suppose another balloon, balloon B, has an initial altitude of 200 feet, and its rate of ascent is given by

$$b(t) = -3t + 24.$$

Give the formula in terms of t for the altitude $B(t)$ of the balloon B after t minutes.

ANSWER: $B(t) =$ _____

7) (16 pts) The following are the graphs of the marginal cost MC and the marginal revenue MR.



Let $TR(q)$ and $TC(q)$ denote the total revenue and total cost, respectively, at q hundred items.

a) Compute $TR(7) - TR(2)$

Answer: _____ hundred \$

b) Find the longest range of quantities within the shown range over which the Total Cost TC is increasing. Justify your answer.

Answer: from $q =$ _____ to $q =$ _____ hundred items

c) Compute $MC'(4)$.

Answer: $MC'(4) =$ _____

d) Estimate $\int_3^6 MR(q) - MC(q) dq$

Answer: _____

e) Suppose $TC(0) = 15$ hundred \$. What is the maximum profit?

Answer: Max profit is _____ hundred \$