

MATH 112 B
Final Exam - Version 1
Winter 2002
Hints and Answers

1. (a) ANSWER: $f'(x) = (x^4 - 3e^x) \left(-2x^{-2} + \frac{1}{x}\right) + \left(\frac{2}{x} + \ln x\right) (4x^3 - 3e^x)$
(b) ANSWER: $\frac{\partial f}{\partial x} = 3(2x)y^{\frac{1}{3}} - 4\left(\frac{1}{2}x^{-\frac{1}{2}}\right)y^3$
(c) ANSWER: $m = -42.60, b = 35.62$
2. (a) ANSWER: $\int \frac{3}{\sqrt{x}} - 5x^7 + 2e^x dx = 6\sqrt{x} - \frac{5}{8}x^8 + 2e^x + C$
(b) ANSWER: $\int_1^5 10x - \frac{3}{5}x^2 dx = 95.2$
3. ANSWER: The best estimate is -65. The integral is clearly negative, which eliminates 60 and 0. Further, if we count the number of complete rectangles “under” the curve, we count 7 full rectangles, each with area 5. So, the area “under” the curve is at least 35. That means that guesses of -13 and -30 aren’t negative enough.
4. (a) HINT: Use the formula for the slope of the secant line. Plug 5 in for t and 0.03 in for r .
ANSWER: $\frac{P(5.03) - P(5)}{0.03} = 16804.06$
(b) HINT: Use the formula for the slope of the secant line. Plug 2 in for t and 5 in for r . This will give a formula for $\frac{P(7) - P(2)}{5}$.
ANSWER: $P(7) - P(2) = 280$
(c) HINT: Use the formula for the slope of the secant line. Plug 0 in for t , 2 in for r , and 20 for $P(0)$. Solve for $P(2)$.
ANSWER: $P(2) = 24$.
(d) HINT: Use the formula for the slope of the secant line. Let r go to 0 to get a formula for $P'(t)$. Plug in 1.2 for t .
ANSWER: $P'(1.2) = 7.33$
5. (a) HINT: Pat’s speed is $P'(t)$. Set $P'(t) = 0$ and solve for t .
ANSWER: $t = 2.76$ seconds
(b) HINT: Pat’s overall average speed is $\frac{P(t)}{t}$. Set $\frac{P(t)}{t} = P'(t)$ and solve for t .
ANSWER: $t = 7.5$ seconds
(c) HINT: Chris’ instantaneous speed is $C'(t)$. You want to know when $C(t)$ is larger than $P(t)$. You may want to think about what the graphs of $C'(t)$ and $P'(t)$ look like.
ANSWER: from $t = 2.42$ seconds to $t = 7.58$ seconds
6. (a) i. ANSWER: F (Profit has a local minimum at $q = 1.5$.)
ii. ANSWER: T (The area under the MR graph from $q = 1.5$ to $q = 11.5$ is larger than the area under the MC graph on that interval.)
iii. ANSWER: F ($C(0) = 6$, but $\int_0^0 c(q) dq = 0$.)

- (b) HINT: Anti-differentiate the formula for MC to get a formula for VC :

$$VC(q) = \frac{1}{3}q^3 - \frac{13}{2}q^2 + 58q + K.$$

Use the fact that $VC(0) = 0$ to find K and plug in 6 for q .

ANSWER: \$186,000

- (c) HINT: Anti-differentiate the formula for MR to get a formula for TR :

$$R(q) = -\frac{1}{3}q^3 + \frac{13}{2}q^2 + 23.5q + K.$$

Since $R(0) = 0$, $K = 0$. Divide $R(q)$ by q to get a formula for $AR(q)$:

$$AR(q) = -\frac{1}{3}q^2 + \frac{13}{2}q + 23.5.$$

Plug in 3 for q .

ANSWER: 40 dollars per item

- (d) HINT: Anti-differentiate the formula for MC to get a formula for TC :

$$C(q) = \frac{1}{3}q^3 - \frac{13}{2}q^2 + 58q + K.$$

Use the fact that $C(0) = 6$ to find the value of K . Plug in $q = 4$.

ANSWER: \$155,333.33

- (e) ANSWER: $-\frac{1}{3}q^3 + \frac{13}{2}q^2 + 23.5q = \frac{1}{3}q^3 - \frac{13}{2}q^2 + 58q + 6$ (or any equivalent form of this equation)