MATH 112 C
Final Exam - Version 1
Winter 2002
Hints and Answers

1. (a) ANSWER: $f'(x) = \dfrac{(\sqrt{x}+\ln x)(2x-e^x)-(x^2-e^x)\left(\frac{1}{2}x^{-1/2}+\frac{1}{x}\right)}{(\sqrt{x}+\ln x)^2}$

   (b) ANSWER: $\dfrac{\partial f}{\partial x} = \dfrac{6}{y}x + \dfrac{4y^3}{x^2}$

   (c) ANSWER: $m = 33.81$, $b = 65.90$

2. (a) ANSWER: $\displaystyle\int \frac{3}{\sqrt{x}} - 5x^7 + 2e^x\ dx = 6\sqrt{x} - \frac{5}{8}x^8 + 2e^x + C$

   (b) ANSWER: $\displaystyle\int_1^5 6x - \frac{3}{25}x^2\ dx = 67.04$

3. ANSWER: $\int_6^9 f(x)\ dx$ is approximately -2 (count squares). $\frac{f(9)-f(0)}{9}$ is the slope of the secant line from the point $(0,2)$ to the point $(9,0)$. This slope is $-\frac{2}{9}$. These are the only two negative options, so they are the smallest. $f'(2)$ is the slope of the tangent line to $f(x)$ at $x = 2$. This slope is 0, so $f'(2)$ comes next. The remaining three are all positive. $\int_0^9 f(x)\ dx$ is $\int_0^6 f(x)\ dx$ minus about 2 squares. $\int_0^5 f(x)\ dx$ is about half a square less than $\int_0^6 f(x)\ dx$. So, from smallest to largest, we have: $\int_6^9 f(x)\ dx$, $\frac{f(9)-f(0)}{9}$, $f'(2)$, $\int_0^9 f(x)\ dx$, $\int_0^5 f(x)\ dx$, and $\int_0^6 f(x)\ dx$.

4. (a) ANSWER: $P(x,y) = 0.43x + 1.05y$ hundred dollars

   (b) ANSWER: $o(x,y) = 0.8x + 0.55y \leq 44$, $t(x,y) = 0.2x + 0.45y \leq 27$

   (c) ANSWER: 63 hundred dollars

   (d) HINT: There's a mistake in the exam. The test should read "If Sunshine produces 33 **hundred** gallons of Ultra Blend and 32 **hundred** gallons of Blend Super Pro..." To compute the amount left over, compute $t(33,32)$ and subtract the result from 27 hundred.
   ANSWER: 6 hundred gallons

5. (a) i. ANSWER: F (Chris ahead of Pat at $t = 1.5$.)

   ii. ANSWER: T (The area under Pat's speed graph from $q = 1.5$ to $q = 11.5$ is larger than the area under Chris' speed graph on that interval.)

   iii. ANSWER: F ($C(0) = 10$, but $\displaystyle\int_0^0 c(t)\ dt = 0$.)

   (b) HINT: You're looking for $C(6) - C(0)$. This is $\displaystyle\int_0^6 t^2 - 13t + 58\ dt$.
   ANSWER: 186 feet

   (c) HINT: Anti-differentiate the formula for $p(t)$ to get a formula for $P(t)$:

$$P(t) = -\frac{1}{3}t^3 + \frac{13}{2}t^2 + 23.5t + K.$$

   Since $P(0) = 0$, $K = 0$. Divide $P(t)$ by t to get a formula for average trip speed $ATS(t)$:

$$ATS(t) = -\frac{1}{3}t^2 + \frac{13}{2}t + 23.5.$$

   Plug in 3 for t.
   ANSWER: 40 feet per second

(d) HINT: Anti-differentiate the formula for $c(t)$ to get a formula for $C(t)$:

$$C(t) = \frac{1}{3}t^3 - \frac{13}{2}t^2 + 58t + K.$$

Use the fact that $C(0) = 10$ to find the value of K. Plug in $t = 4$.
ANSWER: \$159.33

(e) ANSWER: $-\frac{1}{3}t^3 + \frac{13}{2}t^2 + 23.5t = \frac{1}{3}t^3 - \frac{13}{2}t^2 + 58t + 10$ (or any equivalent form of this equation)

6. (a) HINT: Solve $R'(q) = 15$.
ANSWER: $q = 2.76$

(b) HINT: Set $\frac{R(q)}{q} = R'(q)$ and solve for q.
ANSWER: $q = 7.5$ seconds

(c) HINT: You want to know when $MC(q)$ is larger than $MR(q)$. You may want to think about what the graphs of $MC(q)$ and $MR(q)$ look like.
ANSWER: from $q = 2.42$ to $q = 7.58$