MATH 112 B
Final Exam
March 19, 2003

Name _____

Student ID #_____

Section _____

1	15	
2	16	
3	15	
4	19	
5	15	
6	20	
Total	100	

- Check that you have a complete exam. There are six problems, one on each page.
- There are multiple versions of the exam. It will be apparent if you copy someone else's work. Students found engaging in academic misconduct will receive a 0 on this exam.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- When rounding is necessary, you may round your final answer to 2 digits after the decimal.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 1 hour and 50 minutes to complete the exam.

GOOD LUCK!

1. (15 points)

(a) Is $F(x) = x \ln x + 54$ an antiderivative of $f(x) = 1 + \ln x$? Justify your answer.

ANSWER: (circle one) Y N

JUSTIFY:

(b) The best-fitting line for a set of logarithmic data, $z = \ln P(x)$, is $z = -0.5x + 6.7$. Find an exponential formula for $P(x)$. (Write your answer in the form $P(x) = Ae^{rx}$.)

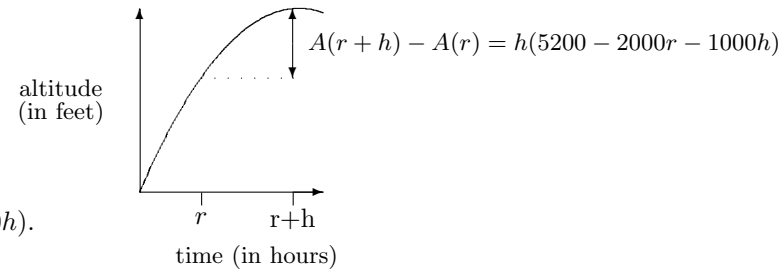
ANSWER: $P(x) =$_____

(c) Compute $\displaystyle\int \frac{1}{4}t^{1/4} - 7 + \frac{1}{t^{1/4}}\ dt$.

ANSWER: $\displaystyle\int \frac{1}{4}t^{1/4} - 7 + \frac{1}{t^{1/4}}\ dt =$_____

2. (16 points) To the right is a rough sketch of a portion of the altitude versus time graph of a balloon. Time is in hours and altitude is in feet. As indicated in the sketch, the change in the altitude of the balloon from time $t = r$ to $t = r + h$ is

$$A(r + h) - A(r) = h(5200 - 2000r - 1000h).$$



(a) The balloon is 4200 feet off the ground at time $t = 1$ hour. How high is the balloon 3 hours later?

ANSWER: _____feet

(b) Find a formula for the (incremental) average rate of ascent from $t = 2$ to $t = 2 + m$.

ANSWER: average rate of ascent = _____

(c) Compute the instantaneous rate of ascent of the balloon at $t = 1.25$ hours.

ANSWER: _____feet per hour

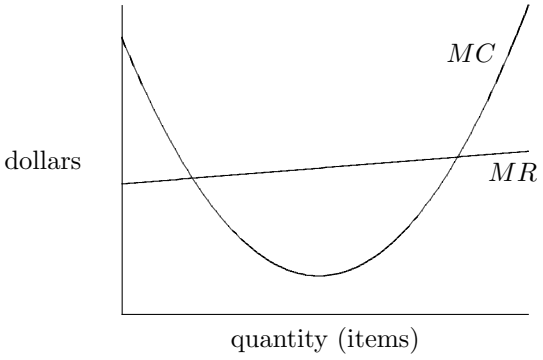
(d) Suppose there is another balloon whose altitude is given by the formula $B(t) = 400t^2 - 1500t + 3400$. Write out a formula in terms of the letter k for the change in altitude of this balloon from time $t = 1$ to $t = 1 + k$. Write your formula in the form ( )k^2 + ( )k + ( ).

ANSWER: change in altitude = ( )k^2 + ( )k + ( )

3. (15 points) The graphs to the right are of Marginal Revenue and Marginal Cost for selling items. The formulas for MR and MC are

$$MR(q) = \frac{2}{25}q + 8 \text{ and}$$

$$MC(q) = \frac{1}{10}q^2 - \frac{121}{50}q + 17.$$



(a) Compute Total Revenue for selling 7 items. (You may assume that $TR(0) = 0$.)

ANSWER: \$_____

(b) Suppose Total Cost for producing 6 items is \$137.89. What is the value of Fixed Cost?

ANSWER: \$_____

(c) At what value of q does the graph of profit have a local minimum? (Round your answer to 2 digits after the decimal — it's OK if the number of items is not a whole number.)

ANSWER: $q =$_____

4. (19 points) You sell Things. The price per Thing is given by the demand function

$$p = h(q) = q^2 - 20q + 100,$$

where q is in hundreds of Things and p is in dollars. Total Revenue (measured in hundreds of dollars) is given by

$$TR(q) = p \cdot q = h(q) \cdot q.$$

The Total Cost (measured in hundreds of dollars) is given by the formula

$$TC(q) = 2q + 1.$$

(a) Give formulas for $TR(q)$ and Profit, $P(q)$.

ANSWER: $TR(q) =$_____

$P(q) =$_____

(b) Find all values of q at which $P(q)$ has a horizontal tangent.

ANSWER: $q =$_____

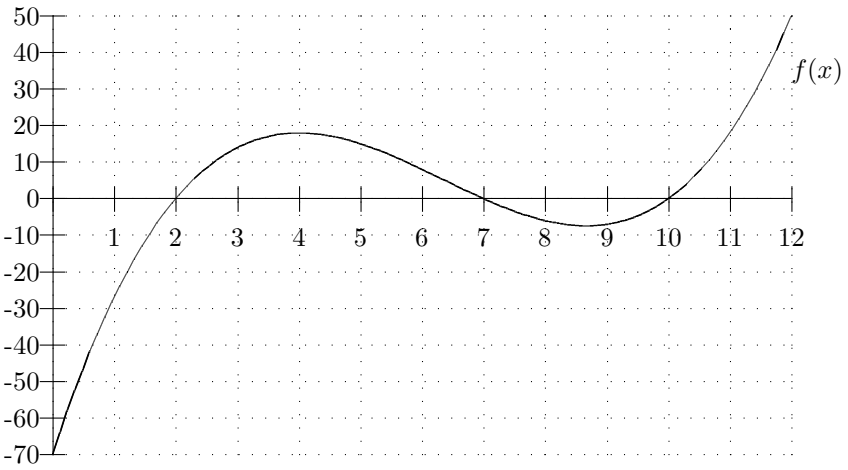
(c) Find the largest possible profit if production is limited to no more than 5 hundred Things.

ANSWER: maximum profit is _____hundred dollars

(d) Find the largest possible profit if production is limited to no more than 3 hundred Things.

ANSWER: maximum profit is _____hundred dollars

5. (15 points) Below is the graph of $f(x)$. Suppose $F(x)$ is an anti-derivative of $f(x)$.



(a) List all values of x where the graph of $F(x)$ has a horizontal tangent.

ANSWER: $x =$_____

(b) List all values of x where the graph of $F''(x)$ crosses the x-axis.

ANSWER: $x =$_____

(c) Give a value of m such that $F''(m) = 5$.

ANSWER: $m =$_____

(d) Suppose $F(0) = 15$. Use the graph to estimate the value of $F(2)$.

ANSWER: $F(2) =$_____

(e) Use the graph to estimate the value of $F'(4)$.

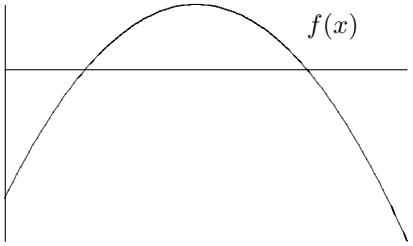
ANSWER: $F'(4) =$_____

(f) Use the graph to estimate the value of $\int_2^5 f(x)\,dx$.

ANSWER: $\int_2^5 f(x)\,dx =$_____

6. (20 points) The graph to the right is the graph of the function

$$f(x) = -x^2 + 19x - 60.$$



Define another function

$$F(m) = \int_0^m f(x)\ dx.$$

(a) Determine all values on the interval from $m = 0$ to $m = 20$ at which $F(m)$ has a local maximum.

ANSWER: $m =$ _____

(b) What is the global minimum value of $F(m)$ on the interval from 0 to 10.?

ANSWER: _____

(c) Suppose you have another function $G(m)$ given by the formula $G(m) = -\frac{1}{4}m^2 + 6m$. Give an interval on which both $F(m)$ and $G(m)$ are increasing.

ANSWER: from $m =$_____to $m =$_____

(d) Give the largest value of $F'(m)$ on the interval from 0 to 20.

ANSWER: _____