MATH 112 B — Winter 2003
Final Exam – Version 1
Hints and Answers

1. (a) HINT: Take the derivative of $F(x)$ to see if $F'(x) = f(x)$. Don't forget to use the product rule.
   ANSWER: Yes.
   (b) ANSWER: $P(x) = 812.41e^{-0.5x}$
   (c) ANSWER: $\int \frac{1}{4}t^{1/4} - 7 + \frac{1}{t^{1/4}}\, dt = \frac{1}{5}t^{5/4} - 7t + \frac{4}{3}t^{3/4} + C$

2. (a) HINT: Take $r = 1$ and $h = 3$. Use the formula to compute $A(4) - A(1)$ then plug in 4200 for $A(1)$ and solve for $A(4)$.
   ANSWER: 4800 feet
   (b) HINT: Take $r = 2$ and $h = m$. Use the formula to compute $A(2 + m) - A(2)$. Then divide by m to get the average rate of ascent.
   ANSWER: 1200-1000m
   (c) HINT: Divide the formula for $A(r + h) - A(r)$ by h and let h go to 0. This gives the formula for $A'(r)$. Plug in 1.25 for r.
   ANSWER: $A'(1.25) = 2700$ feet per hour
   (d) HINTS: You need to compute $B(k + 1) - B(1)$.

$$B(k + 1) = 400(k + 1)^2 - 1500(k + 1) + 3400 = 400k^2 - 700k + 2300$$

$$B(1) = 2300$$

   ANSWER: change in altitude $= 400k^2 - 700k$

3. (a) HINT: TR is an antiderivative of MR: $TR(q) = \frac{1}{25}q^2 + 8q + K$, for some constant K. Since $TR(0) = 0$, $K = 0$ (do you see why?). Plug in 7 for q.
   ANSWER: \$57.96
   (b) TC is an antiderivative of MC: $TC(q) = \frac{1}{30}q^3 - \frac{121}{100}q^2 + 17q + K$, for some constant K. We are told that $TC(6) = \$137.89$. So, set $TC(6) = 137.89$ and solve for K. Then use the fact that Fixed Cost is equal to $TC(0)$.
   ANSWER: \$72.25
   (c) HINT: Look at the graphs of MC and MR. They intersect in two places. The first point of intersection is a place where we go from $MC > MR$ (i.e., profit is decreasing) to $MR > MC$ (i.e., profit is increasing). So, it is at that point of intersection where profit has a local minimum. Set $MC = MR$ and solve for q. This will involve the quadratic formula and you'll get two positive answers. The smallest gives a local min. (What's happening at the other quantity?)
   OR if you don't like that hint, here's another.
   HINT: Use your answers to parts (a) and (b) to find the formula for profit, $P(q)$. Find where $P'(q) = 0$ (i.e., where $P(q)$ has horizontal tangents) and then use the Second Derivative Test to find which of those quantities gives a local min.
   ANSWER: $q = 4.36$

4. (a) ANSWERS: $TR(q) = q^3 - 20q^2 + 100q$, $P(q) = q^3 - 20q^2 + 98q - 1$
   (b) ANSWERS: $q = 3.23$ and 10.10
   (c) HINT: You're looking for the global maximum of $P(q)$ on the interval from $q = 0$ to $q = 5$. Plug $q = 0$, $q = 3.23$ and $q = 5$ into $P(q)$. (Why not $q = 10.10$?)
   ANSWER: maximum profit is 140.58 hundred dollars
   (d) HINT: Now you're looking for the global maximum of $P(q)$ on the interval from $q = 0$ to $q = 3$. Think about how this question is different from part (c).
   ANSWER: maximum profit is 140 hundred dollars

5. BIG HINT: The graph given is that of $f(x)$. This is the *derived graph* of $F(x)$. Questions about $F'(x)$ will be questions about $f(x)$ (the graph given). Questions about $F''(x)$ will be questions about $f'(x)$ (the derived graph of the graph given). Questions about $F(x)$ itself will involve areas "under" the given graph.

   (a) HINT: $F(x)$ has a horizontal tangent when its derived graph crosses the x-axis.
   ANSWER: $x = 2,\ 7,\ 10$

   (b) HINT: The graph of $F''(x)$ is the derived graph of $f(x)$ (the given graph). The graph of $F''(x)$ will cross the x-axis wherever the graph of $f(x)$ has horizontal tangents.
   ANSWER: $x = 4$ and $x = 8.75$ (approximately)

   (c) HINT: Again, $F''(x)$ is the derivative of $f(x)$ (the given graph). That is, $F''(x)$ is the same thing as $f'(x)$. So, you're looking for a number m where $f'(m) = 5$. That is, you're looking for a spot on the graph of $f(x)$ where the tangent line has slope 5. Sketch a line with slope 5 on the graph and find a point on the graph of $f(x)$ where the tangent line is parallel to the line you drew.
   ANSWER: There are two possible answers. $m = 3.3$ or $m = 9.3$ (approximately)

   (d) HINT: $F(x)$ is an antiderivative of $f(x)$. Since $f(x)$ is below the x-axis from 0 to 2, $F(2) = -$(area "under" $f(x)$ from 0 to 2)+(some constant). The value of the constant is $F(0)$, which you are told is 15. Count up the rectangles "under" the graph of $f(x)$ from 0 to 2 and multiply by the area of one rectangle to find the area "under" $f(x)$ from 0 to 2. (You could also compute the area of two trapezoids if you're happier with that.)
   ANSWER: somewhere between -50 and -40

   (e) HINT: Remember that $F'(x)$ and $f(x)$ are the same thing. You're given the graph of $f(x)$. Just find $f(4)$.
   ANSWER: about 18

   (f) HINT: Count up rectangles to approximate the area under $f(x)$ from 2 to 5. (You could also compute the areas of three trapezoids if you'd rather.)
   ANSWER: about 40

6. (a) HINT: The graph given is the graph of $f(x)$, which is the derivative of F. Find where $f(x)$ crosses the x-axis (at $x = 4$ and $x = 15$). F will decrease when f is negative and increase when f is positive. So, F decreases from 0 to 4, increases from 4 to 15 and then decreases from 15 on. What does that mean about local maxima?
   ANSWER: $m = 0$ and 15

   (b) HINT: Use the Fundamental Theorem of Calculus to get a formula for $F(m)$. ($F(m) = -\frac{1}{3}m^3 + \frac{19}{2}m^2 - 60m$) You know from part (a) that F has horizontal tangents at 4 and 15. Plug $m = 0$, $m = 4$ and $m = 10$ (why not 15?) into F to find the global min.
   ANSWER: -109.33

   (c) HINT: You know from part (a) that F is increasing from 4 to 15. Find $G'(m)$ and sketch its graph to see where G is increasing.
   ANSWER: from $m = 4$ to $m = 12$

   (d) HINT: Remember that F' is the same thing as f. So, you're looking for the maximum value of f. The graph of f is a parabola, so you can use the vertex formula. (The vertex of $f(x)$ occurs at $x = 9.5$. So, the maximum value of $f(x)$ is $f(9.5)$.)
   ANSWER: 30.25