MATH 112
Final Exam
March 13, 2004

Name ______________________________

Student ID #______________

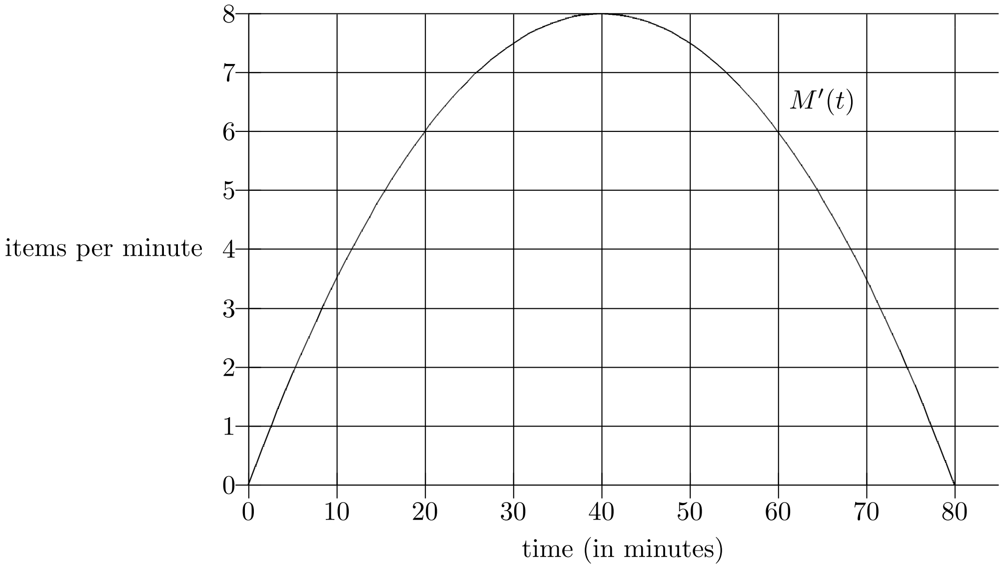
Section ______________

1	14	
2	10	
3	12	
4	17	
5	18	
6	13	
7	16	
Total	100	

- Check that you have a complete exam. Your exam consists of one cover sheet, followed by seven problems on eight pages.
- There are multiple versions of the exam. It will be apparent if you copy someone else's work. Students found engaging in academic misconduct will receive a 0 on this exam.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- When rounding is necessary, you may round your final answer to 2 digits after the decimal.
- We can only give you credit for computations that appear on your exam. Show **all** your work.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 3 hours to complete the exam.

GOOD LUCK!!

1. Freida is taking a learning test in which the time she takes to memorize items from a given list is recorded. Let $M(t)$ be the number of items she can memorize in t minutes. The graph of her *instantaneous learning rate* is shown below. (NOTE: This is *not* the graph of $M(t)$.)



(a) (2 points) Find all times at which the graph of $M(t)$ has a horizontal tangent.

ANSWER: $t =$____________________minutes

(b) (2 points) Find all times at which the graph of $M'(t)$ has a horizontal tangent.

ANSWER: $t =$____________________minutes

(c) (4 points) Estimate the value of $M''(50)$ and use it to determine whether $M(t)$ is concave up or concave down at $t = 50$.

ANSWER: $M''(50) =$____________________; $M(t)$ is concave ____________________at $t = 50$

(d) (3 points) Approximate the value of $\int_{10}^{30} M'(t)\, dt$.

ANSWER: $\int_{20}^{40} M'(t)\, dt =$____________________

(e) (3 points) Approximately how many items can Freida memorize in the first 20 minutes?

ANSWER: ____________________items

2. A Framit manufacturer gathers sales data and determines that price p (in dollars) appears to be an exponential function of quantity q. The manufacturer wishes to find a curve of the form $p = Ae^{mq}$ that best fits the data.

   (a) (5 points) For each price, let $y = \ln p$. Then y is a linear function of q: $y = mq + b$. The mean-squared error function for this line is:

   $$E(b, m) = b^2 + 780m^2 + 44bm - 7.6b - 108m + 17.4.$$

   Compute the line of best fit for the log data.

   ANSWER: ____________________

   (b) (3 points) Use the line you found in part (a) to find a curve of the form $p = Ae^{mq}$ that best fits the manufacturer's data.

   ANSWER: ____________________

   (c) (2 points) Use the exponential function your found in part (b) to predict the total revenue that the manufacturer should expect if 40 Framits are produced and sold.

   ANSWER: ____________________dollars

3. You sell *Items*. You determine that the marginal cost is $MC(q) = 3q^2 - 24q + 48$ (q in items, MC in dollars) and that the marginal revenue is $MR(q) = 120 + 40q - 1.2q^2$ (q in items, MR in dollars).

(a) (3 points) What quantity will yield the largest profit? (Your answer need not be a whole number of items.)

ANSWER: $q =$ ____________________ items

(b) (2 points) Find the formula for total revenue, $TR(q)$. (You may assume that $TR(0) = 0$.)

ANSWER: $TR(q) =$ ____________________________________

(c) (4 points) The total cost of producing 3 Items is \$235. Find the formula for $TC(q)$.

ANSWER: $TC(q) =$ ____________________________________

(d) (3 points) What is the maximum possible profit? (As always, show all work.)

ANSWER: ____________________ dollars

4. A car moves on a long, straight road in such a way that its position (i.e. distance) is given by a function $f(t)$, where t is in seconds and $f(t)$ is in feet. We don't have a formula for $f(t)$, but we know that for all a and h,

$$f(a+h) - f(a) = 2h^2 + 4ah + 5h.$$

(a) (3 points) What is the car's net change in position (how far did the car go) between the times $t = 2$ and $t = 3$?

ANSWER: ____________________feet

(b) (4 points) What is the car's average speed over the interval from $t = 1$ to $t = 4$?

ANSWER: ____________________feet per second

(c) (4 points) If we know that at time $t = 0$ the car's position is $f(0) = 1$, what is $f(4)$?

ANSWER: ____________________feet

(d) (3 points) Compute $f'(1)$.

ANSWER: $f'(t) =$____________________

(e) (3 points) Find a formula for $f'(t)$.

ANSWER: $f'(t) =$________________________________

5. (18 points – 3 points each)

(a) Let $y = 5x^4 - 6x^2 - \dfrac{1}{5x} + \sqrt{x} - \dfrac{1}{\sqrt{x}}$. Find $\dfrac{dy}{dx}$.

(b) Let $f(x, y) = (x^3y^4 - 3xy)^2(4xy^3 - 3x)^5$. Compute the partial derivative $f_y(x, y)$.

(c) Compute the slope of the tangent line to the function $g(x) = \dfrac{x+5}{x-1}$ at the point $x = 2$.

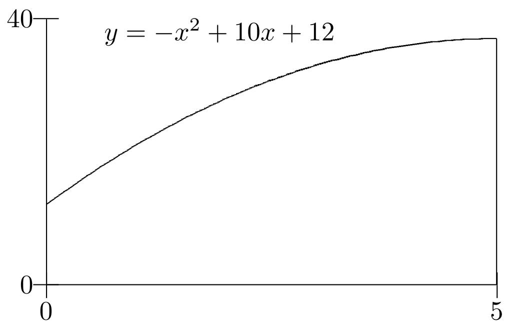
(d) You know the value of a function $h(x)$ at three different values of x.

x	$h(x)$
1	1.5
1.3	1.6
1.5	1.75

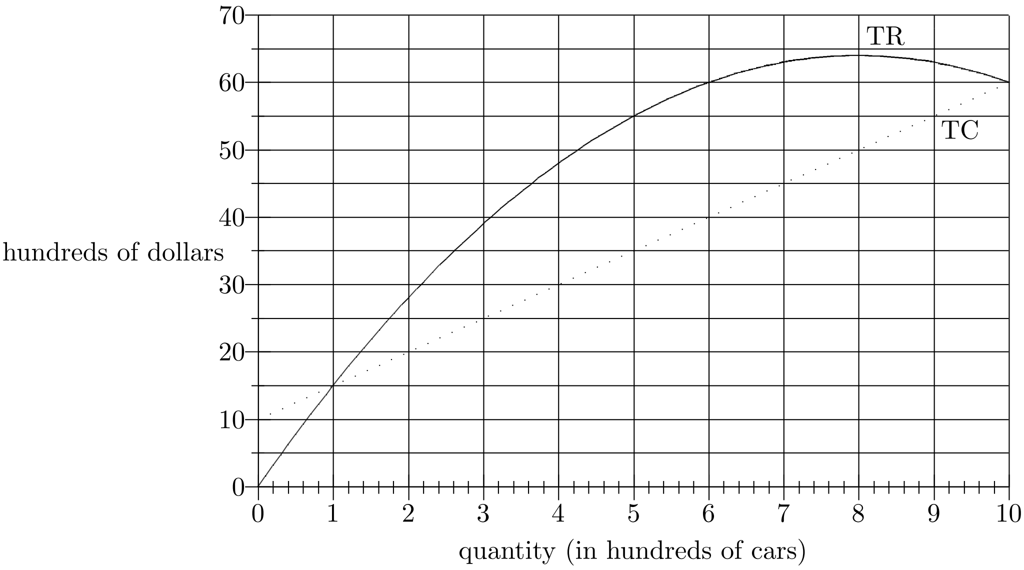
Estimate the value of $h'(1)$.

(e) Evaluate the anti-derivative $\int \left(\frac{2}{5}t^{4/5} - 9 + \frac{1}{\sqrt{t}}\right)\, dt$.

(f) Find the area of the following region.



6. Below are the graphs for total cost and total revenue from running a business that produces and sells toy cars. The quantity is written as q and is in units of hundreds of cars. You are not given formulas for the TC and TR functions.



(a) (3 points) Estimate the cost of the 301st car.

ANSWER: $____________________

(b) (3 points) Are there values of q at which marginal revenue is positive?

ANSWER: (circle one) yes no

If yes, give the largest interval on which marginal revenue is positive.

ANSWER: from $q =$____________________ to $q =$____________________

(c) (3 points) Are there intervals over which marginal revenue is increasing?

ANSWER: (circle one) yes no

If yes, give the largest interval over which marginal revenue is increasing.

ANSWER: from $q =$____________________ to $q =$____________________

(d) (2 points) Estimate the quantity that maximizes profit.

ANSWER: $q =$____________________

(e) (2 points) Give the largest interval over which $MR > MC$.

ANSWER: from $q =$____________________ to $q =$____________________

7. Consider the function $f(x) = x^3 - 12x^2 + 21x + 100$.

(a) (4 points) Find all the values of x at which $f'(x) = 0$.

ANSWER: $x =$ ____________________

(b) (3 points) Use the Second Derivative Test to decide if the smallest x value you found in part (a) is a local maximum or a local minimum.

ANSWER: (circle one)   local max   local min

(c) (3 points) Use calculus to find the global maximum value of $f(x)$ on the interval $[0, 14]$.

ANSWER: ____________________

(d) (3 points) Use calculus to find the global minimum value of $f(x)$ on the interval $[6, 8]$.

ANSWER: ____________________

(e) (3 points) Use calculus to determine where $f(x)$ is concave up.

ANSWER: $f(x)$ is concave up if x is ________________________________