MATH 112 - Winter 2004
Final Exam - Hints and Answers

1. (a) ANSWER: $t = 0,\ 80$ minutes
   (b) ANSWER: $t = 40$ minutes
   (c) HINT: Compute the slope of the tangent line to the graph of $M'(t)$ at $t = 50$.
   ANSWER: $M''(50) \approx -0.1$; $M(t)$ is concave down at $t = 50$.
   (d) HINT: Count rectangles or compute the area of a couple of trapezoids.
   ANSWER: ~ 115
   (e) HINT: Count rectangles or compute the area of a couple of trapezoids.
   ANSWER: ~ 65

2. (a) HINT: $\dfrac{\partial E}{\partial b} = 2b + 44m - 7.6$ and $\dfrac{\partial E}{\partial m} = 1560m + 44b - 108$. Set both partial derivatives equal to 0 and solve for m and b.
   ANSWER: $y = -0.1q + 6$
   (b) HINT: Since $y = \ln p$, you have from part (a) that $\ln p = -0.1q + 6$. Solve for p.
   ANSWER: $p = 403.43e^{-0.1q}$
   (c) HINT: Use the formula from part (b) to compute the price at 40 Framits. Then multiply by 40 Framits to get the total revenue.
   ANSWER: \$295.60

3. (a) HINT: Set MR equal to MC and use the quadratic formula.
   ANSWER: $q = 16.29$ items
   (b) HINT: TR is an antiderivative of MR. Use the fact that $TR(0) = 0$ to find the correct constant of integration.
   ANSWER: $TR(q) = 120q + 20q^2 - 0.4q^3$
   (c) HINT: TC is an antiderivative of MC. Use the fact that $TC(3) = 235$ to find the correct constant of integration.
   ANSWER: $TC(q) = q^3 - 12q^2 + 48q + 172$
   (d) HINT: Compute the profit $(TR - TC)$ at $q = 16.29$ (the answer to part (a)).
   ANSWER: \$3440.67

4. (a) HINT: Let $a = 2$ and $h = 1$ and compute $f(3) - f(2)$.
   ANSWER: 15 feet
   (b) HINT: Let $a = 1$ and $h = 3$ and compute $\dfrac{f(4) - f(1)}{3}$.
   ANSWER: 15 feet per second

(c) HINT: Let $a = 0$ and $h = 4$ and compute $f(4) - f(0)$. You should get that $f(4) - f(0) = 53$. But $f(0) = 1$.
ANSWER: $f(4) = 53$ feet

(d) HINT: Let $a = 1$ and compute $\frac{f(1+h) - f(1)}{h}$. Then let h go to 0.
ANSWER: $f'(1) = 9$

(e) HINT: Let $a = t$ and compute $\frac{f(t+h) - f(t)}{h}$. Then let h go to 0.
ANSWER: $f'(t) = 4t + 5$

5. (a) ANSWER: $\frac{dy}{dx} = 20x^3 - 12x + \frac{1}{5}x^{-2} + \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$

(b) ANSWER: $f_y(x, y) = (x^3y^4 - 3xy)^2 \cdot 5(4xy^3 - 3x)^4(12xy^2) + (4xy^3 - 3x)^5 \cdot 2(x^3y^4 - 3xy)(4x^3y^3 - 3x)$

(c) HINT: $g'(x) = \frac{-6}{(x-1)^2}$
ANSWER: $g'(2) = -6$

(d) ANSWER: There are a lot of acceptable answers to this question. The simplest is to use the slope of the secant line from $x = 1$ to $x = 1.3$. Then $h'(1) \approx \frac{1}{3}$.

(e) ANSWER: $\frac{2}{9}t^{9/5} - 9t + 2\sqrt{t} + K$

(f) HINT: The area is $\int_0^5 -x^2 + 10x + 12\ dx$.
ANSWER: 143.33

6. (a) HINT: The cost of the 301^{st} car is the marginal cost at $q = 3$ hundred cars. Since TC is linear, this is simply the slope of the TC graph.
ANSWER: \$5

(b) ANSWER: yes; from $q = 0$ to $q = 8$

(c) ANSWER: no

(d) HINT: Find the quantity at which the tangent to TR is parallel to TC (i.e. where $MR = MC$).
ANSWER: $q \approx 5.5$ hundred cars

(e) ANSWER: from $q = 0$ to $q =$ your answer from (d)

7. (a) HINT: $f'(x) = 3x^2 - 24x + 21$. Set this equal to 0 and factor or use the quadratic formula to solve for x.
ANSWER: $x = 1,\ 7$

(b) ANSWER: $f''(x) = 6x - 24$; $f''(1) = -18 < 0$. This means f is concave down at $x = 1$ and, thus, f has a local max at $x = 1$.

(c) HINT: The global maximum must occur at $x = 0$, 1, 7, or 14. Compute f at each of these values and pick the largest.

ANSWER: $f(14) = 786$

(d) HINT: The global minimum must occur at $x = 6$, 7, or 8. Compute f at each of these values and pick the smallest.

ANSWER: $f(7) = 2$

(e) HINT: f is concave up wherever $f''(x)$ is positive. $f''(x) = 6x - 24$. Solve the inequality $6x - 24 > 0$ for x.

ANSWER: $f(x)$ is concave up if x is greater than 4.