

Math 112 - Winter 2005
Final Exam
March 12, 2005

Name: _____

Section: _____

Student ID Number: _____

TA's Name: _____

1	16	
2	6	
3	6	
4	12	
5	12	
6	20	
7	12	
8	16	
Total	100	

- **Check that you have 9 pages.**
- Turn your cell phone off and put it away for the duration of the exam.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator when an algebraic method is available, you will not receive full credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- Put your name on your sheet of notes and turn it in with the exam.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 3 hours to complete the exam.

1. (16 points)

(a) Let $f(x) = \frac{2}{3}x^5 - \frac{1}{6}x^3 + xe^{2x} - 5\ln x$. Find $f'(x)$.

(b) Let $z = g(x, y) = x^2y - ye^x + 3x^5 - 5y^3x + 11$. Find $\frac{\partial z}{\partial y}$.

(c) Evaluate:

$$\int \left(x^6 - 3x^4 + \frac{1}{2}x^3 + \frac{2}{x} \right) dx$$

(d) Evaluate:

$$\int_1^3 (x^4 - 6x + 1) dx$$

2. (6 points) Suppose $g(x, y)$ is a function with the following partial derivatives:

$$g_x(x, y) = 3x^2y + y^3 + 8x + y$$

$$g_y(x, y) = y + 3xy^2 + 18y + x$$

Approximate

$$\frac{g(2, 3.0001) - g(2, 3)}{0.0001}.$$

ANSWER: _____

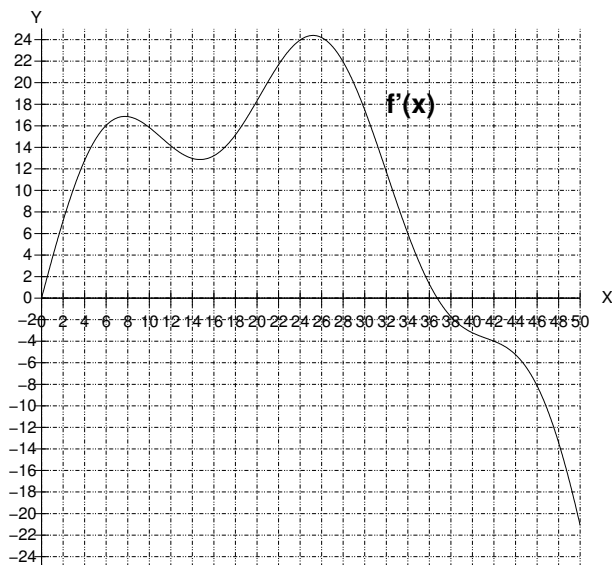
3. (6 points) Is $m(x) = 3e^{4x} - x^2 \ln x + 5$ an anti-derivative of $n(x) = 12e^{4x} - 2x \ln x - x$?

ANSWER: (circle one) YES NO

Why or why not?

4. (12 points) Below is the graph of $f'(x)$, the derivative of $f(x)$. Note that

$$f'(0) = f'(36.8) = 0.$$



- (a) List all intervals on which $f(x)$ is increasing.
- (b) List all intervals on which $f'(x)$ is decreasing.
- (c) List all intervals on which the graph of $f(x)$ is concave up.
- (d) Does $f(x)$ have a local extremum at $x = 36.8$? If so, is it a minimum or a maximum? If not, why not?

5. (12 points) You are trying to find the best fitting line to the population of Portuvania with data from 1970, 1980, 1990, and 2000. The mean-squared error function you get is

$$E(m, b) = b^2 + 350m^2 + 30mb - 482.5b - 7870m + 59008.25.$$

- (a) Find the best fitting line $P(t) = mt + b$ that approximates Portuvania's population T at time t .

ANSWER: $P(t) =$ _____

- (b) Another set of data, for the population of Transyltugal, appears to fit an exponential function. So, you applied the natural logarithm to all of the y -values of your data. You then applied the method of least squares. The resulting line you got was

$$\ln(T(t)) = 1.978 + 0.074t$$

where $T(t)$ is a function that approximates the population of Transyltugal at time t . Set up an equation that you would solve to find when the two countries have the same population. Do not solve it.

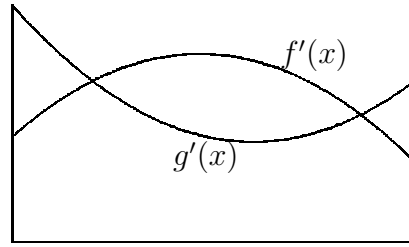
6. (20 points)

The graphs to the right are of $f'(x)$ and $g'(x)$. The formula for $f(x)$ is:

$$f(x) = -\frac{1}{6}x^3 + \frac{7}{2}x^2 + 31x + 20.$$

The formula for $g'(x)$ is:

$$g'(x) = \frac{1}{2}x^2 - 9x + 75.$$



We also know that $g(6) = 350$.

- (a) Find all values of x at which the tangent line to $f(x)$ is parallel to the tangent line to $g(x)$.

ANSWER: $x =$ _____

- (b) What is the global maximum value of $f'(x) - g'(x)$ on the interval from $x = 0$ to $x = 15$?

ANSWER: _____

(This problem is continued on the next page.)

(c) What is the global minimum value of $g(x)$ on the interval from $x = 0$ to $x = 10$?

ANSWER: _____

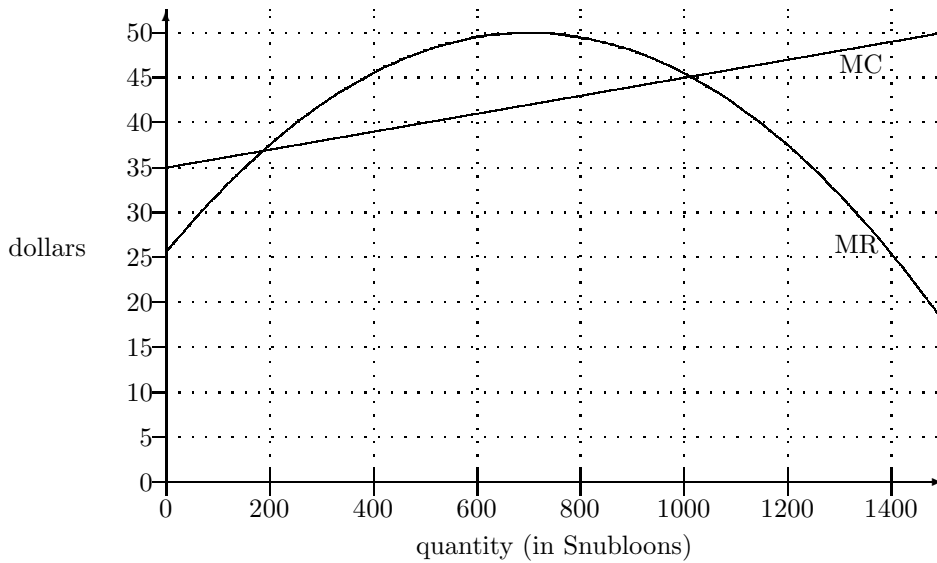
(d) Compute $\int_1^{10} g'(x) dx$.

ANSWER: _____

(e) Compute $\int_0^5 [f'(x) - g'(x)] dx$.

ANSWER: _____

7. (12 points) The graphs of marginal revenue and marginal cost for producing Snubloons are given below. Fixed costs are \$1000.



For each of the following, explain how you arrived at your estimate. Be specific. (For example, “counted rectangles” is not specific; “counted rectangles to compute the area between MR and MC from 1200 to 1400” is specific.)

- (a) Approximate the value of $VC(800) - VC(400)$.

ANSWER: _____

- (b) What is the total revenue if you sell 200 Snubloons?

ANSWER: \$ _____

- (c) Give the largest possible value of profit.

ANSWER: \$ _____

8. (16 points) Two snails, Ben and Jerry, travel along a long, straight path. At $t = 0$, Ben and Jerry are both passing a rock on the path. Let $B(t)$ represent Ben's distance from the rock (in feet) after t hours and let $J(t)$ be Jerry's distance from the rock (in feet) after t hours. Ben's distance is given by

$$B(t) = -t^2 + 10t.$$

Jerry's average speed from $t = k$ to $t = k + h$ is given by

$$\frac{J(k+h) - J(k)}{h} = 3 + 2k + h.$$

- (a) Compute and simplify $\frac{B(t+h) - B(t)}{h}$.

ANSWER: _____

- (b) What is Jerry's average speed from $t = 5$ to $t = 5.01$ hours?

ANSWER: _____ feet per hour

- (c) Give a formula, in terms of r , for the distance Jerry travels from $t = 1$ to $t = r$ hours.

ANSWER: _____

- (d) Find the time at which the snails have the same instantaneous speed.

ANSWER: $t =$ _____ hours