MATH 112 — Winter 2005 Final Exam

Hints and Answers

1. (a) ANSWER: $f'(x) = \frac{10}{3}x^4 - \frac{1}{2}x^2 + e^{2x} + x \cdot e^{2x} \cdot 2 - \frac{5}{x}$

   (b) ANSWER: $\frac{\partial z}{\partial y} = x^2 - e^x - 15y^2x$

   (c) ANSWER: $\frac{1}{7}x^7 - \frac{3}{5}x^5 + \frac{1}{8}x^4 + 2\ln x + C$

   (d) ANSWER: 26.4

2. HINT: $\frac{g(2, 3.0001) - g(2, 3)}{0.0001} \approx g_y(2, 3)$

   ANSWER: $g_y(2, 3) = 113$

3. HINT: Take the derivative of $m(x)$ and see if it's equal to $n(x)$.

   ANSWER: Yes, $m(x)$ is an anti-derivative of $n(x)$ because $m'(x) = n(x)$.

4. (a) HINT: $f(x)$ is increasing wherever $f'(x)$ is positive. You're given the graph of $f'(x)$.
   ANSWER: from $x = 0$ to $x = 36.8$

   (b) ANSWER: from $x \approx 7.5$ to $x \approx 15$ and from $x \approx 25$ to $x = 50$

   (c) HINT: $f(x)$ is concave up wherever $f''(x)$ is positive. $f''(x)$ is positive wherever $f'(x)$ is increasing.
   ANSWER: from $x = 0$ to $x \approx 7.5$ and from $x \approx 15$ to $x \approx 25$

   (d) ANSWER: $f'(x)$ changes from positive to negative at $x = 36.8$. So, $f(x)$ changes from increasing to decreasing at $x = 36.8$. This means that $f(x)$ has a local max at $x = 36.8$.

5. (a) ANSWER: $P(t) = 2.53t + 203.3$

   (b) HINT: $T(t) = e^{1.978+0.074t}$
   ANSWER: $e^{1.978}e^{0.074t} = 2.53t + 203.3$

6. (a) HINT: Compute $f'(x)$, set $f'(x) = g'(x)$, and solve for x.
   ANSWER: $x = 3.53$ and 12.47

   (b) HINT: $f'(x) - g'(x) = -x^2 + 16x - 44$, which is a quadratic whose graph is a parabola that opens downward. The max occurs at the vertex, whose x-coordinate is 8. The global max of $f'(x) - g'(x)$ is $f'(8) - g'(8)$.
   ANSWER: 20

   (c) HINT: We can see from the graph of $g'(x)$ that $g'(x)$ is never 0. In particular, $g'(x)$ is always positive. This means that $g(x)$ is always increasing. So, the global minimum value of $g(x)$ on the interval from $x = 0$ to $x = 10$ occurs at $x = 0$. You therefore need the value of $g(0)$, which requires the formula for $g(x)$. Antidifferentiate $g'(x)$ to get $g(x) = \frac{1}{6}x^3 - \frac{9}{2}x^2 + 75x + K$. You know $g(6) = 350$. Use this fact to find the value of K and then compute $g(0)$.
   ANSWER: 26

   (d) ANSWER: 396

   (e) ANSWER: $-\frac{185}{3} = -61.67$

7. (a) HINT: Compute the area under MC from 400 to 800.
   ANSWER: $\sim 16,400$

(b) HINT: Compute the area under MR from 0 to 200.
ANSWER: ~ 6350

(c) HINT: Profit is maximized at about $q = 1000$. Max profit is (area between MR and MC from 200 to 1000)$-$(area between MR and MC from 0 to 200)$-FC$.
ANSWER: ~ 2750

8. (a) ANSWER: $-2t - h + 10$

(b) ANSWER: 13.01

(c) HINT: Take $k = 1$ and $k + h = r$ in the formula for Jerry's average speed. That is, take $k = 1$ and $r = h - 1$. Then,

$$\frac{J(r) - J(1)}{r - 1} = 3 + 2(1) + (r - 1) = 4 + r.$$

Multiply both sides by $r - 1$ to get distance traveled.
ANSWER: $(4 + r)(r - 1)$ or $r^2 + 3r - 4$

(d) HINT: $B'(t) = -2t + 10$ and $J'(t) = 3 + 2t$ (let h go to 0 in the formula for Jerry's average speed). Set $B'(t) = J'(t)$ and solve for t.
ANSWER: 1.75