MATH 112
Final Exam
March 11, 2006

Name _____

Student ID #_____ Section_____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE: _____

1	12	
2	13	
3	9	
4	11	
5	13	
6	13	
7	12	
8	17	
Total	100	

- Check that your exam contains 8 problems.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (12 points) Compute the indicated derivative. Do not simplify.

   (a) $f(x) = \dfrac{x^2 + 3x + 7}{e^{x^4}(x^3 + 3x)}$

   $f'(x) =$

   (b) $R(q) = (q^2 + 1)(\ln q)\sqrt{7q + 8}$

   $R'(q) =$

   (c) $f(x, y) = x^3y^2 + x^4(y^9 + e^y)$

   $f_x(x, y) =$

2. (13 points) You sell Shimlets. The total revenue and total cost for Shimlets are given by the functions:

$$TR(q) = \frac{1}{3}q^3 - \frac{35}{2}q^2 + 304q \qquad \text{and} \qquad TC(q) = \frac{4}{3}q^3 - 20q^2 + 500q + 1000.$$

(a) Find all values of q at which the *marginal cost*, MC, graph has a horizontal tangent.

ANSWER: $q =$_____

(b) Determine the global maximum value of TR over the interval $q = 1$ to $q = 20$.

ANSWER: global maximum = _____ dollars

(c) Recall that average cost is given by $AC(q) = \dfrac{TC(q)}{q}$.

Use the second derivative to tell whether the average cost (AC) graph is concave up, concave down, or neither at $q = 4$ Shimlets. (If you circle an answer without showing your work and using the second derivative, you will receive no credit.)

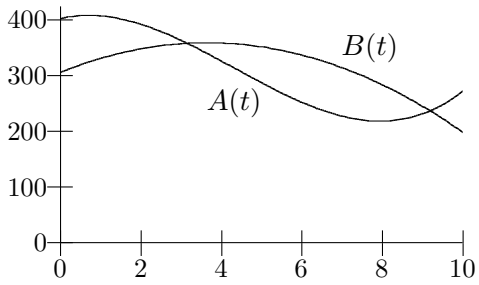
ANSWER: (circle one) CONCAVE UP or CONCAVE DOWN or NEITHER

3. (9 points)

The graph to the right is the *distance* above the ground (in feet) *vs.* time (in minutes) for two balloons. The formulas for these distance graphs are

$$A(t) = t^3 - 13t^2 + 17t + 402 \text{ and}$$

$$B(t) = -4t^2 + 29.3t + 305 \ .$$



(a) Find the height of Balloon A when Balloon B is traveling 18 feet per minute.

ANSWER: _____ feet

(b) Find the *largest* interval on which the derived graph of $A(t)$ is negative and the derived graph of $B(t)$ is negative.

ANSWER: from $t =$ _____ to $t =$ _____ minutes

4. (11 points) You sell Fredbots. The formula for total cost is given by some function $TC(q)$, where q is measured in thousands of Fredbots and TC is in thousands of dollars. We don't know the formula for TC, but we do know that the **change** in total cost from q_1 to q_2 thousand Fredbots is given by:

$$TC(q_2) - TC(q_1) = \frac{4(q_2 - q_1)}{(10 - q_2)(10 - q_1)}.$$

(a) Compute $\dfrac{TC(8) - TC(2)}{6}$.

ANSWER: _____

(b) If you produce 5 thousand Fredbots, your total cost is 2.8 thousand dollars. What is the value of your fixed costs?

ANSWER: $FC =$_____thousand dollars

(c) Find the formula for $TC'(q)$.

ANSWER: $TC'(q) =$_____

5. (13 points) Consider the function

$$f(x, y) = 3xy + \ln(x) + 9y^2.$$

(a) Write out the formulas for $f_x(x, y)$ and $f_y(x, y)$.

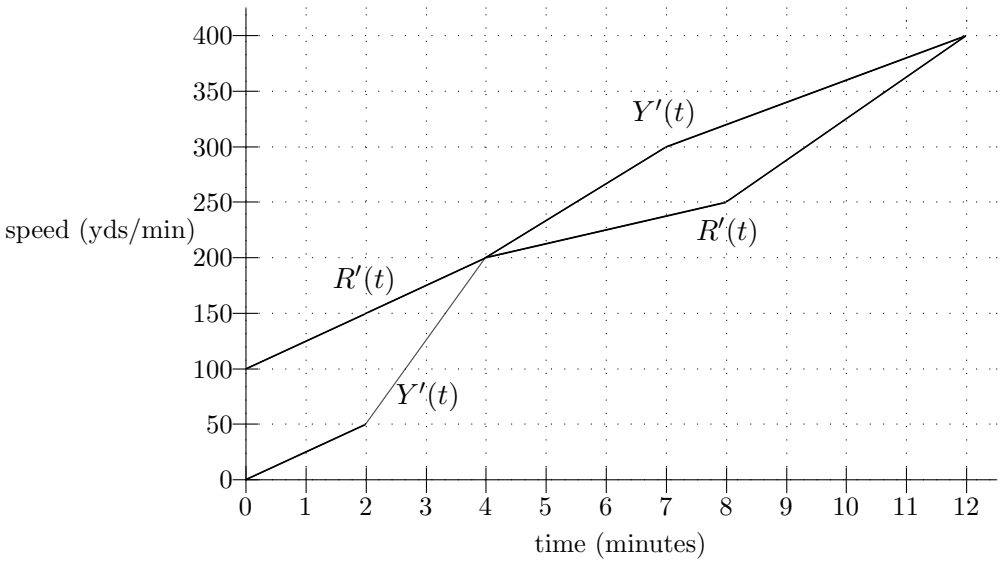
(b) If you fix x to be 2, then $z = f(2, y)$ becomes a function of only one variable, the variable y. Find the slope of the tangent line to the one variable function $f(2, y)$ when $y = 4$.

ANSWER: slope = _____

(c) Find all points (x, y) which are candidates for local minima or local maxima of $f(x, y)$.

ANSWER: list of candidates: (x, y) = _____

6. (13 points) Two moving cars, a red car and a yellow car, are next to one another at time $t = 0$. Their distance formulas are given by two functions $R(t)$ and $Y(t)$. The graph below shows their **instantaneous speed graphs**, $R'(t)$ and $Y'(t)$.



Let $D(t) = R(t) - Y(t)$, the distance by which the red car is ahead of the yellow car.

(a) At what time is the red car ahead of the yellow car by the greatest distance? (You need not show any work.)

ANSWER: $t =$_____minutes

(b) Name a two-minute interval on which $D'(t)$ is positive, then 0, then negative. (You need not show any work.)

ANSWER: from $t =$_____to $t =$_____minutes

(c) How far does the red car travel in the first 4 minutes?

ANSWER: _____yards

(d) Compute the value of $D(4) - D(2)$.

ANSWER: $D(4) - D(2) =$_____

(e) Compute the value of $\int_7^{12} Y'(t)\ dt$.

ANSWER: $\int_7^{12} Y'(t)\ dt =$_____

7. (12 points) Two electronically controlled cars (Car A and Car B) are moving along parallel tracks and are next to one another at time $t = 0$. You are given the following information about them:

$$\begin{array}{ll} \text{Distance vs. time for Car A:} & A(t) = \frac{1}{6}t^3 - t^2 + 14t + 10 \\ \text{Speed vs. time for Car B:} & B'(t) = 3t + 11 \end{array}$$

Time, t, is given in seconds and distance is given in feet.

(a) Note that $A(0) = B(0) = 10$. Write out the formula for $B(t)$. There should be no undetermined constant in the formula.

ANSWER: $B(t) =$ _____

(b) Find *all* times that the speed of Car A is the same as the speed of Car B.

ANSWER: _____

(c) Evaluate $\int_1^5 A'(t)dt$.

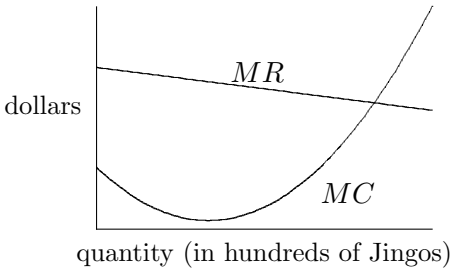
ANSWER: $\int_1^5 A'(t)dt =$ _____

8. (17 points)

To the right are the graphs of marginal revenue and marginal cost for producing Jingos. Their formulas are:

$MR(q) = -4q+226$ and $MC(q) = 3q^2-30q+87$,

where q is measured in hundreds of Jingos and MR and MC are both in dollars.



(a) What quantity will yield the largest total revenue?

ANSWER: $q =$_____hundred Jingos

(b) Recall that variable cost (in hundreds of dollars) is given by an antiderivative of $MC(q)$ and that $VC(0) = 0$. Give the formula for $VC(q)$.

ANSWER: $VC(q) =$_____

(c) If $TC(5) = 608$, find the value of fixed costs.

ANSWER: $FC =$_____hundred dollars

(d) Find the quantity at which profit reaches its maximum value.

ANSWER: $q =$_____hundred Jingos

(e) Assuming the same fixed costs as in part (c), what is the largest possible profit?

ANSWER: _____hundred dollars