MATH 112 – FINAL EXAM Hints and Answers
Winter 2006

1. (a) ANSWER: $f'(x) = \dfrac{\left[e^{x^4}(x^3+3x)\right](2x+3) - (x^2+3x+7)\left[e^{x^4}(3x^2+3) + (x^3+3x)\cdot e^{x^4}\cdot(4x^3)\right]}{\left[e^{x^4}(x^3+3x)\right]^2}$

   (b) ANSWER: $R'(q) = \left[(q^2+1)(\ln q)\right]\cdot\frac{1}{2}(7q+8)^{-1/2}(7) + \sqrt{7q+8}\left[(q^2+1)\cdot\frac{1}{q} + (\ln q)(2q)\right]$

   (c) ANSWER: $f_x(x,y) = 3x^2y^2 + 4x^3(y^9+e^y)$

2. (a) HINT: Compute $MC'(q)$, set it equal to 0, and solve for q.
   ANSWER: $q = 5$

   (b) HINT: Find the critical numbers of $TR(q)$ ($q = 16$ and $q = 19$). Plug the critical numbers and endpoints into $TR(q)$. Choose the largest value of TR.
   ANSWER: 1749.33 dollars

   (c) HINT: $AC''(q) = \frac{8}{3} + \frac{2000}{q^3}$.
   ANSWER: CONCAVE UP

3. (a) HINT: Set $B'(t)$ equal to 18 and solve for t: $t = 1.4125$. Then compute $A(1.4125)$.
   ANSWER: 402.89 feet

   (b) HINT: $B'(t) = -8t + 29.3$ and $A'(t) = 3t^2 - 26t + 17$. $B'(t)$ is a line with a positive "y"-intercept and negative slope. Its t-intercept is $t = 3.6625$ (set $B'(t) = 0$ and solve for t). So, $B'(t)$ is negative from $t = 3.6625$ on. $A'(t)$ is a quadratic function whose graph is a parabola that opens upward. Its t-intercepts are $t = 0.7124$ and $t = 7.9543$ (set $A'(t) = 0$ and use the quadratic formula to solve for t). So, $A'(t)$ is negative from $t = 0.7124$ to $t = 7.9543$.
   ANSWER: from $t = 3.6625$ to $t = 7.9543$

4. (a) HINT: Use the formula given with $q_2 = 8$ and $q_1 = 2$ to compute $TC(8) - TC(2)$. Then divide by 6.
   ANSWER: $\frac{1}{4}$

   (b) HINT: Recall that fixed cost is given by $TC(0)$. Use the formula given with $q_2 = 5$ and $q_1 = 0$ to compute $TC(5) - TC(0)$. Then substitute the value 2.8 for $TC(5)$ and solve for $TC(0)$.
   ANSWER: 2.4 thousand dollars

   (c) HINT: Use the formula given with $q_2 = q + h$ and $q_1 = q$ to compute $TC(q+h) - TC(q)$. Divide this formula by h. You now have the formula for the slope of a secant line: $\dfrac{TC(q+h) - TC(q)}{h}$. Let h go to 0 to get the formula for $TC'(q)$.
   ANSWER: $TC'(q) = \frac{4}{(10-q)^2}$

5. (a) ANSWER: $f_x(x,y) = 3y + \frac{1}{x}$; $f_y(x,y) = 3x + 18y$

   (b) HINT: $f(2,y) = 6y + \ln 2 + 9y^2$. The slope of the tangent line to this function at $y = 4$ is $f_y(2,4)$.
   ANSWER: 78

   (c) HINT: Set $f_x(x,y)$ and $f_y(x,y)$ equal to 0 and solve the resulting system of equations.
   ANSWER: $(1.41, -0.24)$ and $(-1.41, 0.24)$

6. (a) HINT: The red car is ahead by the greatest distance when the cars have the same speed.
   ANSWER: $t = 4$ minutes

(b) HINT: The distance between the cars is increasing from $t = 0$ to $t = 4$ and then it begins to decrease. So, $D'(t)$ will be positive from $t = 0$ to $t = 4$ and then will be negative for a while.

ANSWER: Any two-minute interval that contains the time $t = 4$ will work.

(c) HINT: Compute the area under the graph of $R'(t)$ from $t = 0$ to $t = 4$.

ANSWER: 600 yards

(d) HINT: Compute the area between $R'(t)$ and $Y'(t)$ from $t = 2$ to $t = 4$.

ANSWER: 100

(e) HINT: Compute the area under $Y'(t)$ from $t = 7$ to $t = 12$.

ANSWER: 1750.

7. (a) HINT: $B(t)$ is an anti-derivative of $B'(t)$. So, $B(t) = \frac{3}{2}t^2 + 11t + K$, for some constant K. $B(0)$ then is equal to K and equal to 10. This means $K = 10$.

ANSWER: $B(t) = \frac{3}{2}t^2 + 11t + 10$

(b) HINT: Compute $A'(t)$, set $A'(t)$ equal to $B'(t)$ and solve the resulting quadratic equation for t.

ANSWER: $t = 0.64$ and $t = 9.36$

(c) HINT: $\int_1^5 A'(t)\,dt = A(5) - A(1)$

ANSWER: 52.67

8. (a) HINT: MR is the derivative of TR and MR is positive for a while and then negative. This means that TR is increasing for a while and then decreasing. TR will be largest when MR is equal to 0. So set $MR(q)$ equal to 0 and solve for q.

ANSWER: $q = 56.5$ hundred Jingos

(b) ANSWER: $VC(q) = q^3 - 15q^2 + 87q$

(c) HINT: $TC(q) = VC(q) + FC$. So, $TC(5) = VC(5) + FC$. Use the formula for $VC(q)$ that you found in part (b) to find that $VC(5) = 185$. Then $608 = 185 + FC$.

ANSWER: $FC = 423$ hundred dollars

(d) HINT: Profit is maximized when MR equals MC. Set $MR(q)$ equal to $MC(q)$ and solve for q.

ANSWER: $q = 12.4$ hundred Jingos

(e) HINT: You have a formula for $TC(q)$ from parts (b) and (c). The formula for $TR(q)$ can be obtained by finding the anti-derivative of $MR(q)$. Then maximum profit will be $TR(12.4) - TC(12.4)$.

ANSWER: 1392.86 hundred dollars