

MATH 112
Final Exam
March 10, 2007

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	14		6	10	
2	12		7	10	
3	10		8	12	
4	10		9	10	
5	12		Total	100	

- Your exam should consist of 9 problems. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- Unless otherwise indicated, you may round your FINAL ANSWER to two digits after the decimal.
- You may use a calculator for arithmetic purposes only (such as plugging into the quadratic formula or plugging into a function). ALL other work must be written and demonstrated on your exam. No credit will be given for guess-and-check or calculator methods, even if they give the correct answer.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam.

GOOD LUCK!

1. (14 points) The total revenue and total cost for selling Frumtops are:

$$TR(q) = -5q^2 + 80q \text{ and } TC(q) = q^3 - 12q^2 + 60q + 60,$$

where q is in thousands of Frumtops and TR and TC are both in thousands of dollars.

- (a) Use the derivative rules to find formulas for marginal revenue and marginal cost.

$$\text{ANSWER: } MR(q) = \underline{\hspace{10em}}$$

$$MC(q) = \underline{\hspace{10em}}$$

- (b) What does it cost to produce the 6,001st Frumtop?

$$\text{ANSWER: } \underline{\hspace{10em}} \text{ dollars}$$

- (c) What quantity maximizes profit?

$$\text{ANSWER: } q = \underline{\hspace{10em}} \text{ thousand Frumtops}$$

- (d) Find all quantities at which the slope of the tangent line to the marginal revenue graph is the same as the slope of the tangent line to the marginal cost graph.

$$\text{ANSWER: } q = \underline{\hspace{10em}} \text{ thousand Frumtops}$$

- (e) Is the graph of **total revenue** concave up, concave down, or neither at $q = 5$? (As always, you must show your work.)

$$\text{ANSWER: (circle one) } \quad \text{concave up} \quad \text{concave down} \quad \text{neither}$$

2. (12 points) Let $y = f(x) = x^2 - 5x + 2\ln(x) + 6$

(a) Find all the critical numbers of $f(x)$.

ANSWER: $x =$ _____

(b) Use the Second Derivative Test to determine whether each critical number you found in part (a) gives a local minimum or a local maximum, or state that the test is inconclusive.

(c) Find the the global maximum and global minimum values of $f(x)$ on the interval from $x = 1$ to $x = 10$.

ANSWERS: GLOBAL MINIMUM: _____

GLOBAL MAXIMUM: _____

3. (10 points) This week it is your job to buy muffins and bagels for a meeting at your office. Each muffin will feed only 1 person, while each bagel will be cut in half to feed 2 people.

A muffin costs \$1.50 and a bagel costs \$2.00. Your boss gives you \$30.00 to spend. When you get to the bakery they only have 12 bagels left.

Let x be the number of muffins that you buy and y be the number of bagels you buy. You want to maximize the number of people that you can feed.

- (a) Give the constraints and the objective function for this problem.

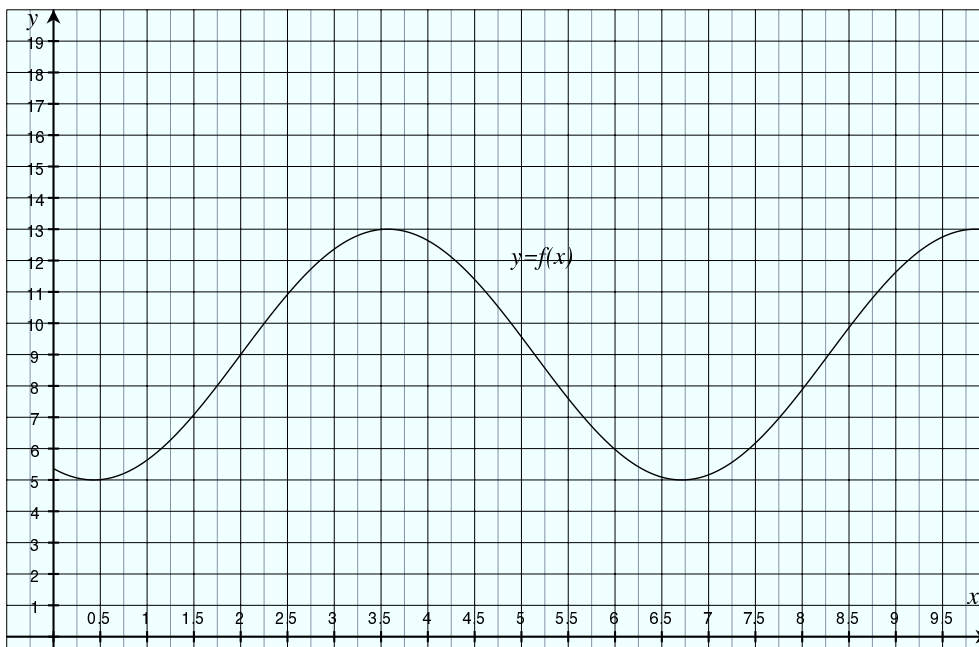
ANSWER: objective function: _____
constraints: _____

- (b) Sketch the feasible region, clearly label all vertices with their exact values.

- (c) How many muffins, x , and bagels, y , should you buy to maximize the number of people you can feed? Justify your answer.

ANSWER: $x =$ _____ muffins
 $y =$ _____ bagels

4. (10 points) The graph below is of the function $y = f(x)$.



(a) Give two values of x at which $f'(x) = 2$.

ANSWER: $x =$ _____ and $x =$ _____

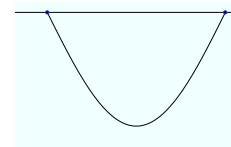
(b) Approximate the value of $\frac{f(4+h) - f(4)}{h}$ if $h = 0.001$.

ANSWER: $\frac{f(4+h) - f(4)}{h} =$ _____

(c) Give an interval of length 1 over which the graph of $f'(x)$ is positive and decreasing.

ANSWER: from $x =$ _____ to $x =$ _____

(d) Give an interval over which the graph of $f'(x)$ is shaped like this:



ANSWER: from $x =$ _____ to $x =$ _____

5. (12 points) Two balloons are next to each other at time $t = 0$. You are given the following information about them:

Altitude vs. time: $A(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 50$

for balloon A

Rate of ascent: $A'(t) =$

for balloon A

Altitude vs. time $B(t) =$

for balloon B:

Rate of ascent $B'(t) = 20 - 10t$

for balloon B:

where t is in minutes and Altitude is in feet.

- (a) Find the highest altitude Balloon A reaches in the first 3 minutes.

ANSWER: highest altitude = _____ feet

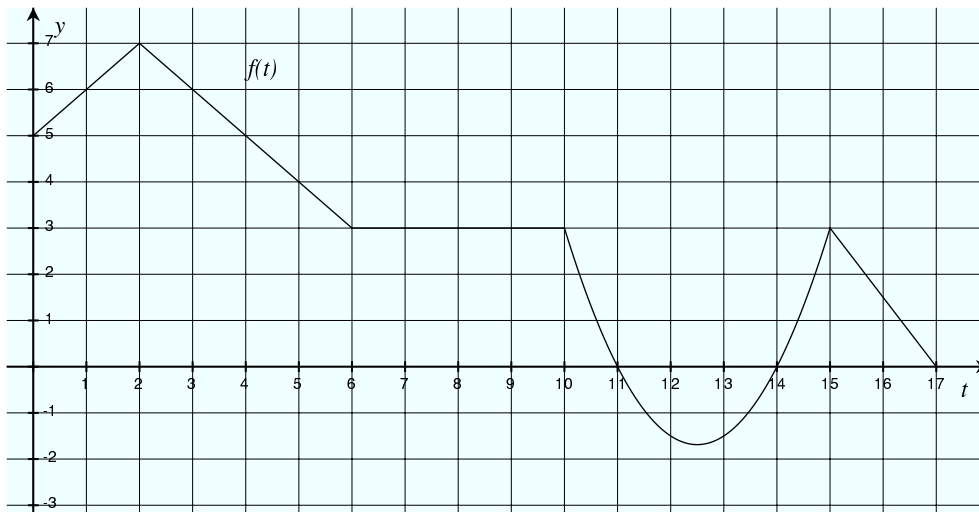
- (b) Use the fact that $B(0) = A(0)$ to determine the formula for $B(t)$ without any undetermined constants.

ANSWER: $B(t) =$ _____

- (c) Find all times, t , at which balloon B is ascending 3 ft per minute faster than balloon A .

ANSWER: $t =$ _____ minutes

6. (10 points)

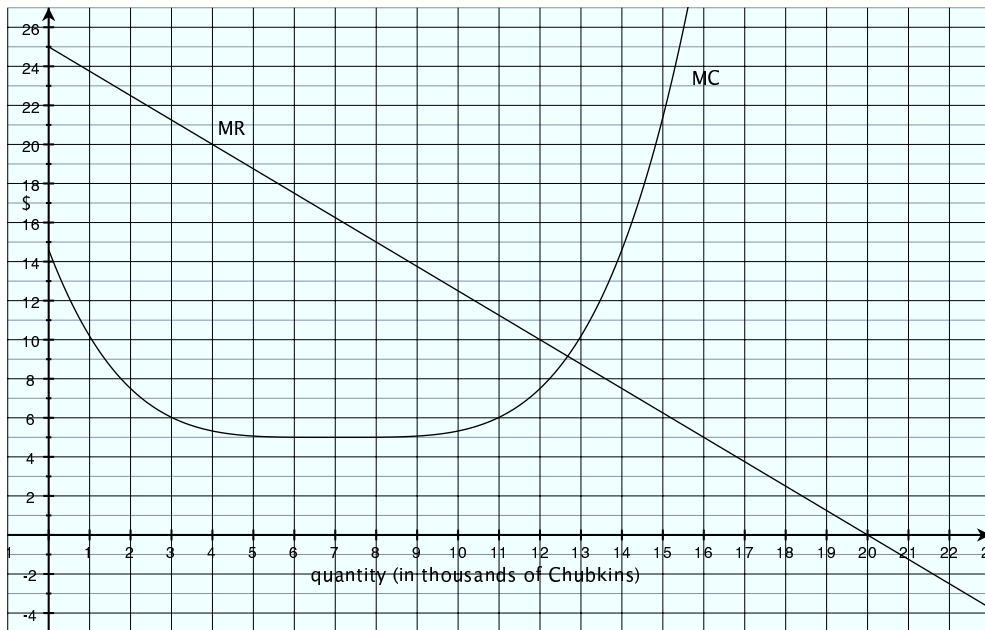
The graph below is of the function $y = f(t)$.

Using the graph above, we define a new function

$$A(m) = \int_0^m f(t) dt.$$

(a) Find the value of t between 0 and 17 at which $f(t)$ has a global maximum.ANSWER: $t =$ _____(b) Find all values of m between 0 and 17 at which $A(m)$ has a local minimum.ANSWER: $m =$ _____(c) Determine the value of $f'(4)$.ANSWER: $f'(4) =$ _____(d) Determine the value of $A'(4)$.ANSWER: $A'(4) =$ _____(e) Compute $\int_6^{10} f(t) dt$.ANSWER: $\int_6^{10} f(t) dt =$ _____

7. (10 points) The graphs below are of marginal revenue (MR) and marginal cost (MC) (both in **dollars**) for producing q **thousand** Chubkins.



- (a) Determine the increase in total revenue if q increases from 12 to 20 thousand Chubkins.

ANSWER: _____ thousand dollars

- (b) If q changes from 6 to 8 thousand Chubkins, does profit increase or decrease? By how much?

ANSWER: Profit (circle one) increases decreases by _____ thousand dollars.

- (c) What is the largest value of total revenue?

ANSWER: _____ thousand dollars

- (d) If fixed costs are \$11.2 thousand dollars, what is the profit earned for selling 2 thousand Chubkins?

ANSWER: _____ thousand dollars

8. (12 points)

(a) Evaluate: $\int_1^2 x^3 + 4x - 1 \, dx$

(b) Evaluate: $\int 3x + \frac{4}{x} - \sqrt{x} \, dx$

(c) Let $z = \frac{t^2 + e^{5t+1}}{t^3 - 4}$. Find $\frac{dz}{dt}$. Do not simplify.

(d) Let $f(x, y) = 3x^3y + x^2 \ln(x) + y^2$. Find $f_x(x, y)$.

9. (10 points) Let $z = f(x, y) = -3x^2 + 2y^2 + 36x + 12xy$.

(a) Write out the formulas for $f_x(x, y)$ and $f_y(x, y)$.

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

(b) Use partial derivatives to tell which of the following numbers is bigger:

$$\mathbf{A} = \frac{f(1.0001, 1) - f(1, 1)}{0.0001} \quad \text{or} \quad \mathbf{B} = \frac{f(1, 1.0001) - f(1, 1)}{0.0001}$$

Justify your answer. (That is, you must show work to get credit.)

ANSWER: (circle one) **A** **B**

(c) Find all points (x, y) which are candidates for local maxima or local minima of $f(x, y)$ or show that there are no such candidates.

ANSWER: (Check one) The candidate(s) is/are: $(x, y) = \underline{\hspace{10cm}}$
 There are no such candidates.