MATH 112 – FINAL EXAM Hints and Answers
Winter 2007

1. (a) ANSWER: $MR(q) = -10q + 80$, $MC(q) = 3q^2 - 24q + 60$
   (b) HINT: Compute $MC(6)$.
   ANSWER: 24 dollars
   (c) HINT: Set $MR(q) = MC(q)$ and solve for q.
   ANSWER: $q = 5.81$ thousand Frumtops
   (d) HINT: Set $MR'(q) = MC'(q)$ and solve for q.
   ANSWER: $q = 2.333$ thousand Frumtops
   (e) ANSWER: $TR''(q) = -10$, which is less than 0 for all values of q. In particular, $TR''(5) < 0$, which implies that TR is concave down at $q = 5$.

2. (a) HINT: $f'(x) = 2x - 5 + \frac{2}{x}$. Set $f'(x) = 0$ and solve for x.
   ANSWER: $x = \frac{1}{2}$ and $x = 2$
   (b) HINT: $f''(x) = 2 - \frac{2}{x^2}$. Plug in the values of x from part (a) and determine the sign of f'' at each value.
   ANSWER: $x = \frac{1}{2}$ gives a local max and $x = 2$ gives a local min.
   (c) HINT: Evaluate $f(x)$ at the endpoints $x = 1$ and $x = 10$ and at the only critical point between those x-values: $x = 2$.
   ANSWER: Global min: $f(2) = 1.37$; Global max: $f(10) = 60.61$

3. (a) ANSWER: objective function: $F(x, y) = x + 2y$; constraints: $1.5x + 2y \leq 30$ and $y \leq 12$
   (b) ANSWER: The vertices are $(0, 0)$, $(0, 12)$, $(4, 12)$, and $(20, 0)$.
   (c) ANSWER: $x = 4$ muffins and $y = 12$ bagels

4. (a) HINT: Sketch a reference line with slope 2 and find the points on the graph of $f(x)$ at which the tangent line is parallel to the reference line.
   ANSWER: $x = 1$, 3, 7.25, and 9.25 are all possible approximate answers
   (b) HINT: Draw the tangent line to $f(x)$ at $x = 4$ and compute its slope.
   ANSWER: approximately -1.76
   (c) ANSWER: Your answer should be an interval of length 1 that begins and ends between $x = 2$ and $x = 3.5$ or between $x = 8.25$ and $x = 9.75$.
   (d) ANSWER: (approximate) from $x = 3.5$ to $x = 6.75$

5. (a) HINT: Maximize the function $A(t)$ on the interval from $t = 0$ to $t = 3$. The critical numbers of A are $t = 2$ and $t = 5$. So, plug your endpoints $t = 0$ and $t = 3$ and the critical number $t = 2$ into $A(t)$ and choose the largest value.
   ANSWER: 58.67 feet
   (b) ANSWER: $B(t) = -5t^2 + 20t + 50$
   (c) HINT: Set $B'(t) - A'(t) = 3$ and solve for t.
   ANSWER: $t = 1.54$ minutes

6. (a) ANSWER: $t = 2$
   (b) ANSWER: $m = 14$
   (c) ANSWER: $f'(4) = -1$

(d) ANSWER: $A'(4) = 5$

(e) ANSWER: 12

7. (a) HINT: Compute the area under MR from $q = 12$ to $q = 20$.
ANSWER: 40 thousand dollars

(b) ANSWER: Since $MR > MC$ on the interval from $q = 6$ to $q = 8$, profit increases on this interval. The amount that profit increases is the area between MR and MC on this interval: approximately 22.5 thousand dollars.

(c) HINT: Total revenue is largest when MR crosses the q-axis at $q = 20$. The largest value of TR is therefore the area under the MR graph from $q = 0$ to $q = 20$.
ANSWER: 250 thousand dollars

(d) HINT: The profit at $q = 2$ will equal the area between MR and MC from $q = 0$ to $q = 2$ minus the fixed costs.
ANSWER: approximately 15.3 thousand dollars

8. (a) ANSWER: $\frac{35}{4}$

(b) ANSWER: $\frac{3}{2}x^2 + 4\ln x + \frac{2}{3}x^{3/2} + K$

(c) ANSWER: $\dfrac{dz}{dt} = \dfrac{(t^3-4)(2t+5e^{5t+1}) - (t^2+e^{5t+1})(3t^2)}{(t^3-4)^2}$

(d) ANSWER: $f_x(x,y) = 9x^2y + x^2 \cdot \frac{1}{x} + (\ln x)(2x)$

9. (a) ANSWER: $f_x(x,y) = -6x + 36 + 12y$, $f_y(x,y) = 4y + 12x$

(b) ANSWER: $A \approx f_x(1,1) = 42$ and $B \approx f_y(1,1) = 16$. So, A is bigger.

(c) HINT: Set f_x and f_y equal to 0 and solve the resulting system of equations.
ANSWER: $x = \frac{6}{7}$ and $y = -\frac{18}{7}$