MATH 112

Final Exam

March 14, 2009

Name ____

Student ID #___________________ Section ___________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________________

1	20	
2	10	
3	10	
4	13	
5	18	
6	13	
7	16	
Total	100	

- Your exam should consist of this cover sheet, followed by seven problems on ten pages. Check that you have a complete exam.
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. Clearly label lines and points that you are using, shade areas, and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method or read a value from a graph on your calculator when an algebraic method is available, you may not receive full credit.

GOOD LUCK!!

1. (20 points)

   (a) Compute the indicated derivative. DO NOT SIMPLIFY.

      i. $g(t) = \dfrac{e^{(t^2-3t)}}{(2t+1)^{1/5}}$

         $g'(t) =$

      ii. $m(v) = \ln(\sqrt{v}e^{v^3} - v^2)$

         $m'(v) =$

      iii. $P = (5x^3 + x\ln y)(x^2y + xy^4)$

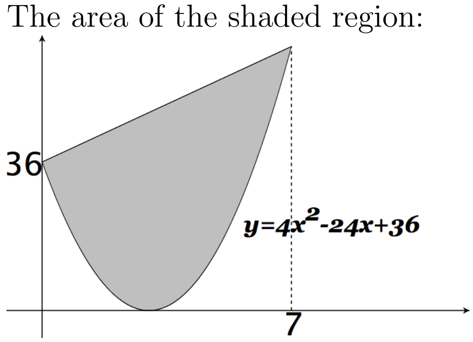
         $\dfrac{\partial P}{\partial y} =$

(b) Compute each of the following.

i. $\int \left(\frac{3}{4t} - \frac{6}{\sqrt[3]{t}} + \frac{1}{\sqrt[3]{t^2}} \right) dt$

ii. $\int_1^9 9x^2 - \frac{2}{3\sqrt{x}}\, dx$

iii. The area of the shaded region:



2. (10 points) You sell Things and the **Total Revenue** (in dollars) for selling q Things is given by the formula

$$TR(q) = \frac{1}{6}q^4 - \frac{55}{6}q^3 + 150q^2 + 100q.$$

(a) Recall that **Average Revenue** is $AR(q) = \frac{TR(q)}{q}$. Find all values of q at which **Average Revenue** has a horizontal tangent line.

ANSWER: $q =$ ______________________________

(b) Find the global minimum and maximum values of **Marginal Revenue** on the interval from $q = 5$ to $q = 10$. (As always, show all your work.)

ANSWER: minimum = ______________; maximum = ______________

(c) Is **Total Revenue** concave up or concave down at $q = 15$? (As always, show your work.)

ANSWER: (circle one) concave up   concave down

3. (10 points) You make and sell two types of shoes: oxfords and ankle boots. Each pair of oxfords requires 15 ounces of leather and 12 ounces of rubber. Each pair of ankle boots requires 20 ounces of leather and 14 ounces of rubber. Each month, you are supplied with 60,600 ounces of leather and 45,480 ounces of rubber. Each pair of oxfords yields \$7 profit and each pair of ankle boots yields \$8 profit.

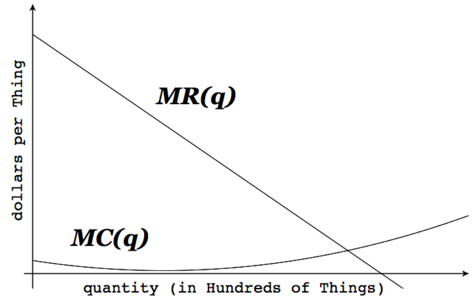
   Let x represent the number of pairs of oxfords you sell in a month and y represent the number of pairs of ankle boots you sell in a month. Find the values of x and y that maximize profit.

ANSWER: $x =$ __________________ pairs of oxfords

$y =$ __________________ pairs of ankle boots

4. (13 points)

You sell Things. The formulas for **Marginal Revenue** and **Marginal Cost** are graphed to the right and are given by
$MR(q) = 180 - 5q$
$MC(q) = 0.03q^2 - 0.9q + 8.75$,
where q is measured in **Hundreds of Things** and MR and MC are in **dollars per Thing.** You also know your fixed costs: $FC = 4$ Hundred Dollars.



(a) Compute the change in **Total Revenue** that results from increasing quantity from 1 to 5 hundred Things.

ANSWER: __________________Hundred Dollars

(b) Compute the **Profit** you earn if you produce 15 Hundred Things.

ANSWER: __________________Hundred Dollars

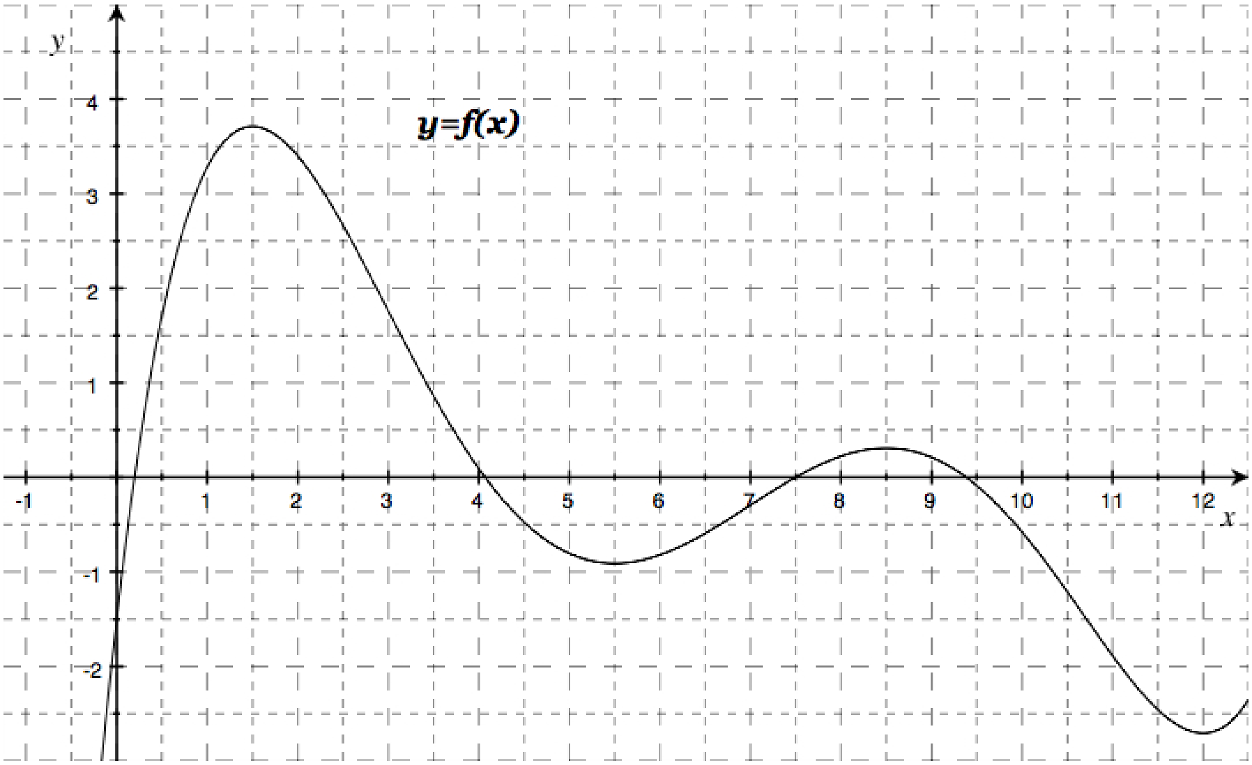
(c) Compute the quantity that yields the largest profit.

ANSWER: __________________Hundred Things

(d) Recall that **Average Cost** is $AC(q) = \dfrac{TC(q)}{q}$. Find the slope of the tangent line to $AC(q)$ at $q = 10$.

ANSWER: __________________

5. (18 points) Below is the graph of a function $y = f(x)$.



Define the function $A(m)$ by $A(m) = \int_0^m f(x)\,dx$.

<u>NOTE:</u> You do **not** need to show any work for the problems **on this page.**

(a) Name all values of m at which $A(m)$ has a local minimum.

ANSWER: $m =$ ______________________________

(b) Give the one-minute interval over which $A(m)$ increases the most.

ANSWER: from _______________ to _______________

(c) True or False?

circle one

T **F** $A(2.51) > A(2.50)$

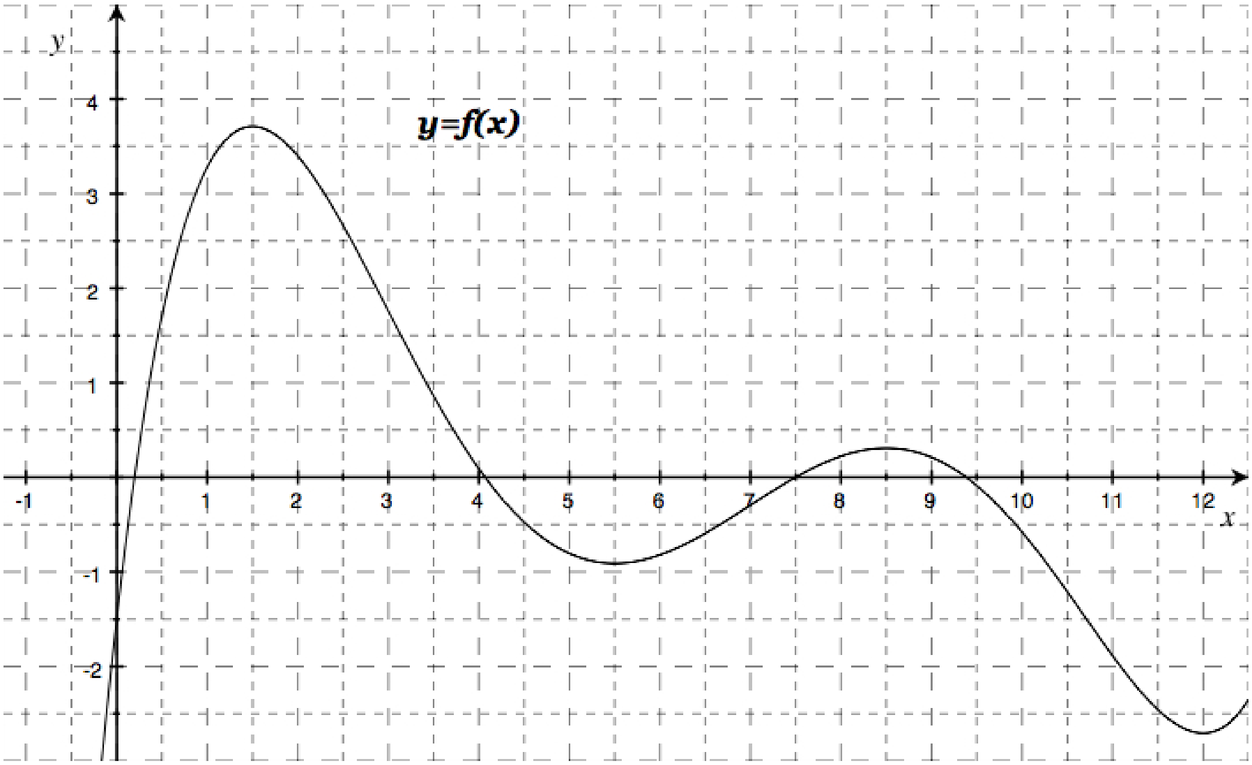
T **F** $f(2.51) > f(2.50)$

T **F** $A(10.01) > A(10.00)$

T **F** $f'(1.00) > f'(1.01)$

(THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.)

Here is the graph of $y = f(x)$ again.



And, again, $A(m) = \int_0^m f(x)\,dx$.

<u>NOTE:</u> The problems on this page **require some justification**: clearly mark points and lines on the graph, shade areas, show calculations of slopes and areas, etc.

(e) Compute $A(1)$.

ANSWER: $A(1) =$ ________________

(f) Compute $A'(12)$.

ANSWER: $A'(12) =$ ________________

(g) Compute $A''(5)$.

ANSWER: $A''(5) =$ ________________

(h) Name a value of x at which $f(x) = f(7)$.

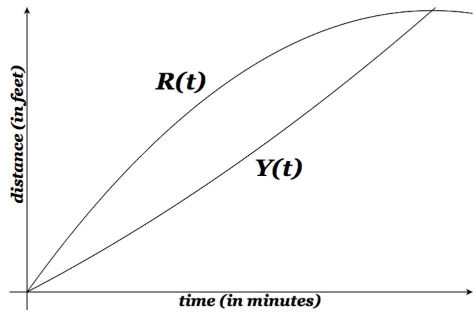
ANSWER: $x =$ ________________

(i) Compute $A(4) - A(2)$.

ANSWER: $A(4) - A(2) =$ ________________

6. (13 points)

The graphs at right show the distance vs. time graphs for two electronically controlled cars, one red and one yellow. The formulas for these graphs are:
$R(t) = 90t - 3t^2$ and $Y(t) = t^2 + 30t$.



(a) Give a formula, in terms of a and h, for the average speed of the Red car over the h-minute interval beginning at $t = a$. Simplify your answer as much as possible.

ANSWER: average speed = ____________________

(b) Find the longest interval over which the instantaneous speed of the Red car exceeds the instantaneous speed of the Yellow car.

ANSWER: from $t =$____________ to $t =$____________

(c) Find all times at which the Yellow car is traveling 6 feet per minute faster than the Red car.

ANSWER: $t =$____________________

7. (16 points) Water flows in and out of two vats, vat A and vat B. The **rate of flow** (in gallons per minute) for vat A at time t minutes is given by the formula:

$$a(t) = 6t^2 - 66t + 144,$$

while the **amount** (in gallons) in vat B at time t minutes is given by the formula:

$$B(t) = 2t^2 - 30t + 65.$$

At $t = 0$, vat A **contains exactly 60 gallons more than** vat B.

(a) Find the longest interval on which the water level in vat A is decreasing.

ANSWER: from $t =$__________________ to $t =$__________________ minutes

(b) Determine all times at which the water level in vat A is reaching a local minimum value.

ANSWER: $t =$__________________

(c) Write out the formula for the amount $A(t)$ in vat A at time t.

ANSWER: $A(t) =$____________________________________

(THIS PROBLEM CONTINUES ON THE NEXT PAGE.)

Here are those formulas again.

The **rate of flow** (in gallons per minute) for vat A at time t minutes is given by the formula:

$$a(t) = 3t^2 - 39t + 108,$$

while the **amount** (in gallons) in vat B at time t minutes is given by the formula:

$$B(t) = 2t^2 - 24t + 80.$$

(d) What is the highest rate at which water is flowing into vat B on the interval from $t = 7$ to $t = 10$?

ANSWER: ____________________gallons per minute.

(e) How much water flows into vat A from $t = 1$ to $t = 3$?

ANSWER: ____________________gallons