MATH 112 – FINAL EXAM Hints and Answers
Version Beta
Winter 2009

1. (a) i. ANSWER: $g'(t) = \dfrac{(2t+1)^{1/5} \cdot e^{(t^2-3t)} \cdot (2t-3) - e^{(t^2-3t)} \cdot \frac{1}{5}(2t+1)^{-4/5}(2)}{[(2t+1)^{1/5}]^2}$

   ii. ANSWER: $m'(v) = \dfrac{1}{(\sqrt{v}e^{v^3} - v^2)} \cdot \left[\sqrt{v} \cdot e^{v^3} \cdot 3v^2 + e^{v^3} \cdot \dfrac{1}{2\sqrt{v}} - 2v\right]$

   iii. ANSWER: $\dfrac{\partial P}{\partial y} = (5x^3 + x\ln y)(x^2 + 4xy^3) + (x^2y + xy^4)\left(\dfrac{x}{y}\right)$

   (b) i. ANSWER: $\dfrac{3}{4}\ln t - 9t^{2/3} + 3t^{1/3} + K$

   ii. ANSWER: 2181.33

   (c) HINT: The line that bounds the top of the shaded region has the equation $y = 4x + 36$. So, the area is equal to $\int_0^7 (4x+36) - (4x^2 - 24x + 36)\,dx$.
   ANSWER: 228.67

2. (a) HINT: $AR(q) = \frac{1}{6}q^3 - \frac{55}{6}q^2 + 150q + 100$. Compute $AR'(q)$, set it equal to 0, and solve the resulting equation.
   ANSWER: $q = 12.32$ and 24.34

   (b) HINT: $MR(q) = \frac{2}{3}q^3 - \frac{55}{2}q^2 + 300q + 100$. Use the recipe to find the global max and min of this function on the interval from $q = 5$ to $q = 10$.
   ANSWER: minimum=995.83, maximum=1084.375

   (c) HINT: Determine the sign of $TR''(15)$.
   ANSWER: concave down

3. ANSWER: $x = 3790$ pairs of oxfords, $y = 0$ pairs of ankle boots

4. (a) HINT: $TR(5) - TR(1) = \int_1^5 MR(q)\,dq$
   ANSWER: 660 Hundred Dollars

   (b) HINT: $TR(15) = \int_0^{15} MR(q)\,dq$, $VC(15) = \int_0^{15} MC(q)\,dq$, and profit at $q = 15$ is equal to $TR(15) - TC(15) = TR(15) - [VC(15) + FC]$.
   ANSWER: 2069.75

   (c) HINT: Set MR equal to MC and solve the resulting equation.
   ANSWER: 33.54 Hundred Things

   (d) HINT: $TC(q) = 0.01q^3 - 0.45q^2 + 8.75q + 4$ and $AC(q) = 0.01q^2 - 0.45q + 8.75 + \frac{4}{q}$. Compute $AC'(10)$.
   ANSWER: -0.29

5. (a) ANSWER: $m = 0.2$, 7.5

   (b) ANSWER: from 1 to 2

   (c) ANSWER: T, F, F, T

   (d) ANSWER: $A(1) \approx 1.28125$

   (e) ANSWER: $A'(12) \approx -2.7$

   (f) ANSWER: $A''(5) \approx -2/3$

(g) ANSWER: $x \approx 0.2$, 4.4, OR 9.75

(h) ANSWER: $A(4) - A(2) \approx 3.5$

6. (a) ANSWER: average speed $= 90 - 6a - 3h$

   (b) HINT: Compute $R'(t)$ and $Y'(t)$ and sketch their graphs.
   ANSWER: from $t = 0$ to $t = 7.5$

   (c) HINT: Set $Y'(t) - R'(t)$ equal to 6 and solve the resulting equation.
   ANSWER: $t = 8.25$

7. (a) HINT: You want to know where the graph of vat A's **rate of flow** is negative. Sketch the graph of $a(t)$ and determine where it is *below* the t-axis.
   ANSWER: from $t = 3$ to $t = 8$ minutes

   (b) HINT: The water **level** hits a local min when its **rate** changes from negative to positive. Look at the sketch of $a(t)$ that you made in part (a).
   ANSWER: $t = 8$

   (c) HINT: $A(t) = 2t^3 - 33t^2 + 144t + K$, for some constant K. At $t = 0$, vat B contains 65 gallons and vat A contains 60 gallons more than that. That is, $A(0) = 125$.
   ANSWER: $A(t) = 2t^3 - 33t^2 + 144t + 125$

   (d) HINT: Vat B's **rate of flow** is a *linear* function with a positive slope. Therefore, it is highest at the right-hand endpoint of the interval. Compute $B'(10)$.
   ANSWER: 16 gallons per minute

   (e) HINT: $A(3) - A(1) = \int_1^3 a(t)\,dt$.
   ANSWER: 86 gallons