MATH 112
Final Exam
March 12, 2011

Name ______________________________

Student ID #______________ Section ______________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

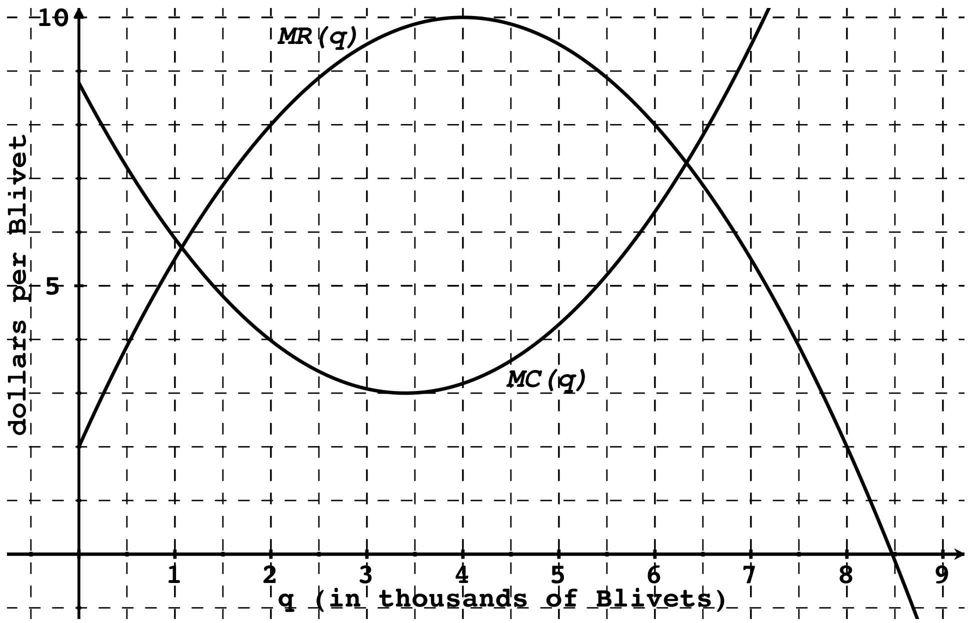
SIGNATURE:______________________________

1	18	
2	22	
3	16	
4	10	
5	12	
6	12	
7	10	
Total	100	

- Your exam should consist of this cover sheet, followed by 7 problems on 8 pages. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- Unless otherwise indicated, you must use the methods of this course and show all of your work. The correct answer with little or no supporting work may result in no credit. If you use a guess-and-check method or read a value from a graph on your calculator when an algebraic method is available, you may not receive full credit. On questions with graphs, clearly mark lines and points and/or shade areas and clearly show all calculations.
- There are multiple versions of this exam. You've signed an honor statement. Don't cheat.

GOOD LUCK!!

1. (18 points) You sell Blivets. Below are the graphs of marginal revenue and marginal cost for producing Blivets. You are not given the formulas for $MR(q)$ and $MC(q)$.



(a) Compute the value of $TR(3.5) - TR(3)$.

ANSWER: $TR(3.5) - TR(3) =$ ____________________ thousand dollars

(b) Compute $VC(2)$.

ANSWER: $VC(2) =$ ____________________ thousand dollars

(c) Your fixed costs are 3 thousand dollars. Compute profit at $q = 2$ thousand dollars.

ANSWER: ____________________ thousand dollars

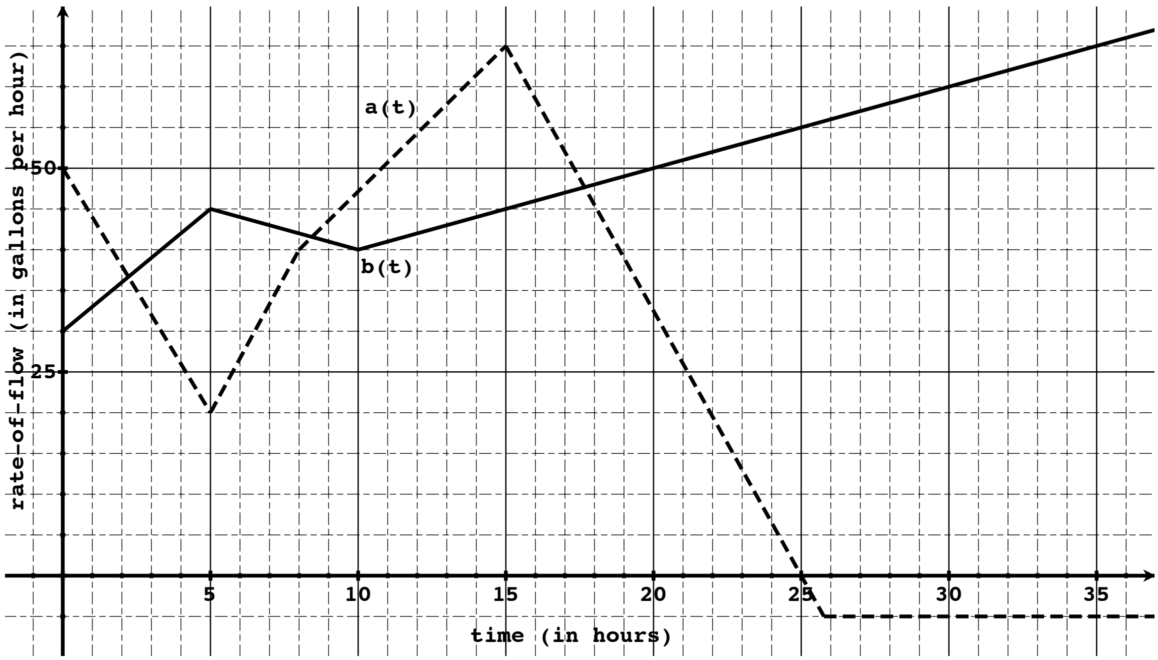
(d) Give the largest interval on which the **derivative of profit** is positive.

ANSWER: from $q =$ ____________________ to $q =$ ____________________

(e) Give the largest value of $MC'(q)$ on the interval from $q = 4$ to $q = 5.5$.

ANSWER: ____________________

2. (22 points) The graphs below are of rate-of-flow of water in and out of two vats. The dashed graph is the rate-of-flow for Vat A and the solid graph is the rate-of-flow for Vat B.



Let:

- $A(t)$ = the amount of water in Vat A at time t
- $B(t)$ = the amount of water in Vat B at time t
- $a(t)$ = the rate-of-flow of Vat A at time t
- $b(t)$ = the rate-of-flow of Vat B at time t

At $t = 0$, Vat A contains 25 gallons and Vat B contains 10 gallons.

(a) How much water is in Vat A at $t = 5$?

ANSWER: ____________________gallons

(b) Compute $A'(8) - A'(5)$.

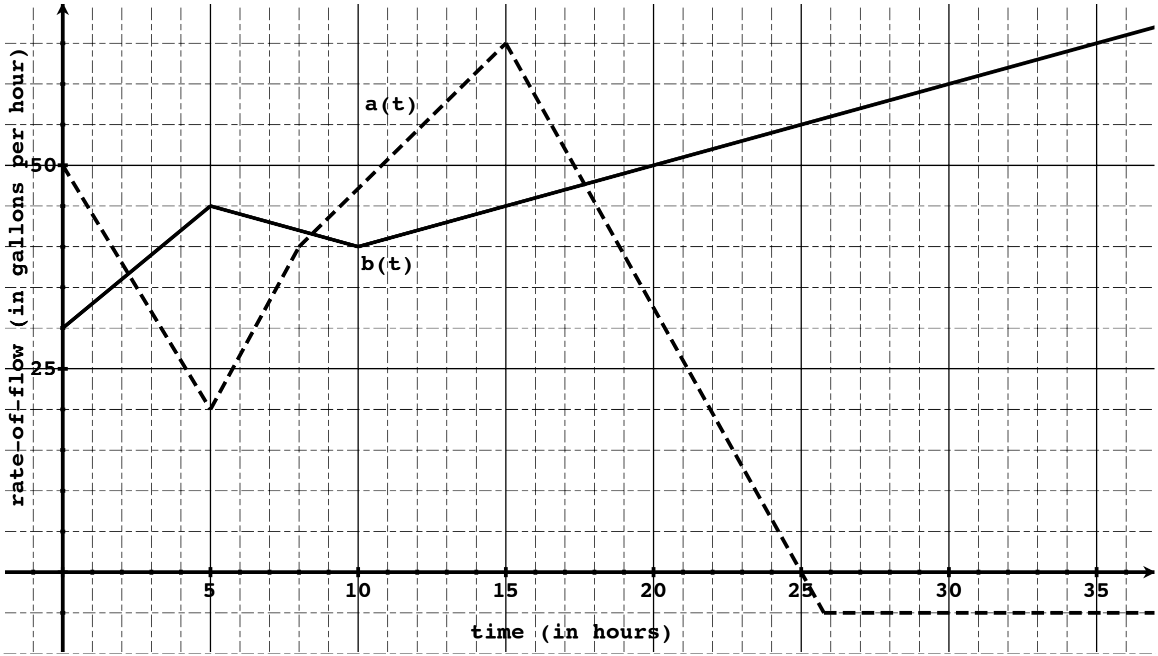
ANSWER: ____________________gallons per hour

(c) Compute $B''(20)$.

ANSWER: $B''(20) =$____________________

(THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.)

Here are those graphs again.



(d) Let $D(t) = A(t) - B(t)$. Name all intervals shown on which $D(t)$ is decreasing.

(e) Give the global maximum value of $A'(t)$ on the interval from $t = 0$ to $t = 37$. Include units with your answer.

ANSWER: ____________________UNITS: ____________________

(f) Give the global maximum value of $A(t)$ on the interval from $t = 0$ to $t = 37$. Include units with your answer.

ANSWER: ____________________UNITS: ____________________

(g) Give a two-hour interval on which $B(t)$ decreases and then increases or explain why no such interval exists.

3. (16 points)

   (a) Compute the derivative. Do not simplify.

      i. $y = xe^{\sqrt{x}} \ln(x^5 - 4x)$

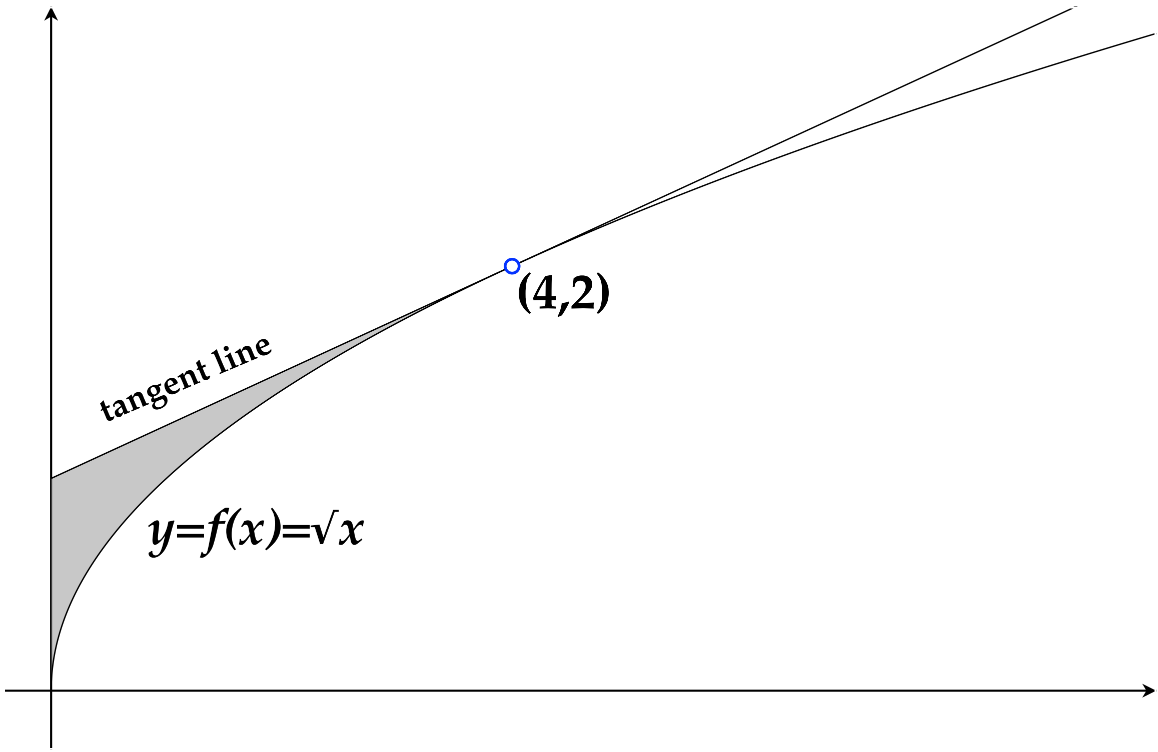
      ii. $z = \sqrt{\dfrac{4t^3 - 6t}{5t^2 + 1}}$

   (b) Compute the integral.

      i. $\displaystyle\int u^{3/2} + \frac{6}{5\sqrt{u}} - \frac{1}{7u}\, du$

      ii. $\displaystyle\int_1^2 x^3 - x\, dx$

4. (10 points) The graph below shows the graph of $y = f(x) = \sqrt{x}$ and the line tangent to $f(x)$ at the point $(4, 2)$.



(a) Find the equation of the tangent line shown.

(b) Compute the area of the shaded region.

5. (12 points) Suppose $f(x, y) = -5x^2 - 5x - 3y^2 + 4y + 10xy + 18$.

   (a) Find all pairs (x, y) which are candidates for a local maximum or a local minimum of $f(x, y)$. Show all work.

   ANSWER: ____________________

   (b) Use partial derivatives to determine which of these numbers is bigger: A or B? Show all work.

$$A = \frac{f(4, 6.0001) - f(4, 6)}{0.0001} \qquad B = \frac{f(4.0001, 6) - f(4, 6)}{0.0001}.$$

   ANSWER: (circle one) A B is bigger

   (c) Consider the functions $f(1, y)$, $f(3, y)$, $f(6, y)$, and $f(8, y)$. Which has the steepest graph at $y = 1$? Show all work.

   ANSWER: The function with the steepest graph at $y = 1$ is: ____________________

6. (12 points) Suppose distance traveled by an object is given by

$$f(t) = -4t^2 + 40t,$$

where t is in minutes and distance is in miles.

(a) Write out a formula for distance traveled from $t = 2$ to $t = 2 + h$. Simplify.

(b) Find the formula for the instantaneous speed of the object at time t.

(c) The distance traveled by a second object is given by a function $g(t)$, where t is in minutes and distance is in miles. We don't know the formula for $g(t)$ but we know that

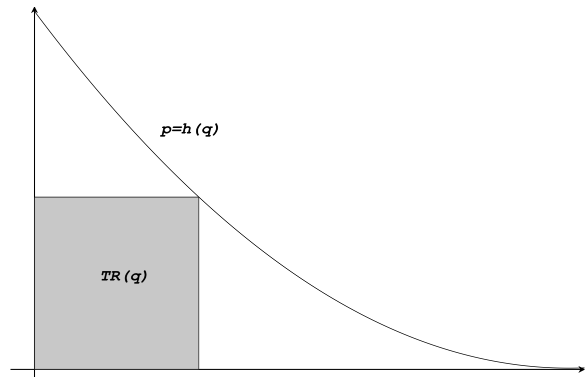
$$g(t + h) - g(t) = -10th - 5h^2 + 60h.$$

How far does this second object travel from $t = 2$ to $t = 6$?

ANSWER: ____________________miles

(d) Find the instantaneous speed of the second object at time t.

7. (10 points)



The demand curve for selling Trinkets is given at right. The price per Trinket (in dollars) if you sell q Trinkets is given by the function

$$p = h(q) = q^2 - 142.5q + 5250.$$

(a) Give the formula for total revenue, $TR(q)$.

(b) Production is limited so that you can produce no more than 50 Trinkets. Find all critical numbers of $TR(q)$ between $q = 0$ and $q = 50$ and use the Second Derivative Test to classify each one as a local maximum or a local minimum.

(c) Again, production is limited so that q must be between 0 and 50 Trinkets. At what price should you sell Trinkets in order to maximize total revenue?

ANSWER: ____________________dollars per Trinket