MATH 112 — WINTER 2011
Final Exam — Hints and Answers

1. (a) HINT: Compute the area under MR from 3 to 3.5.
   ANSWER: ~ 4.825 thousand dollars
   (b) HINT: Compute the area under MC from 0 to 2.
   ANSWER: ~ 12.8 thousand dollars
   (c) HINT: Profit at 2 is (the area under MR from 0 to 2) minus (the area under MC from 0 to 2) minus (fixed costs).
   ANSWER: ~ -4.4 thousand dollars
   (d) HINT: The derivative of profit is positive when profit is increasing and profit increases when MR is greater than MC.
   ANSWER: from $q \approx 1.1$ to $q \approx 6.3$
   (e) HINT: Use your ruler to see that MC' increases on the interval from $q = 4$ to $q = 5.5$. Draw the tangent line to MC at $q = 5.5$ and compute its slope.
   ANSWER: ~ 2.15

2. (a) HINT: Compute the area under $a(t)$ from 0 to 5 and add the initial 25 gallons.
   ANSWER: 200 gallons
   (b) HINT: $A'(8) - A'(5) = a(8) - a(5)$
   ANSWER: 20 gallons per hour
   (c) HINT: Compute the slope of the tangent line to $B'(t) = b(t)$ at $t = 20$.
   ANSWER: $B''(20) = 1$
   (d) HINT: $D(t)$ is decreasing when $D'(t)$ is negative. $D'(t) = A'(t) - B'(t) = a(t) - b(t)$. This is negative when the graph of $a(t)$ is below the graph of $b(t)$.
   ANSWER: from 2.1 to 8.3 and from 17.8 to 37
   (e) HINT: $A'(t) = a(t)$. So, the largest value of $A'(t)$ is simply the y-coordinate of the highest point on the graph of $a(t)$.
   ANSWER: 65 gallons per hour
   (f) HINT: $A(t)$ increases as long as $a(t)$ is positive. So, $A(t)$ will be reaching its maximum when $a(t)$ changes from positive to negative: at $t = 25$. Compute the area under $a(t)$ from 0 to 25 and then add on the initial 25 gallons (since at $t = 0$, vat A contains 25 gallons).
   ANSWER: 982.5 gallons
   (g) ANSWER: No such interval exists since $b(t)$ is always positive, which means $B(t)$ is always increasing.

3. (a) i. ANSWER: $\frac{dy}{dx} = (xe^{\sqrt{x}})\left(\frac{5x^4 - 4}{x^5 - 4x}\right) + \ln(x^5 - 4x)\left[xe^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + e^{\sqrt{x}}\right]$
   ii. ANSWER: $\frac{dz}{dt} = \frac{1}{2}\left(\frac{4t^3 - 6t}{5t^2 + 1}\right)^{-1/2}\left[\frac{(5t^2+1)(12t^2-6) - (4t^3-6t)(10t)}{(5t^2+1)^2}\right]$
   (b) i. ANSWER: $\frac{2}{5}u^{5/2} + \frac{12}{5}u^{1/2} - \frac{1}{7}\ln u + K$
   ii. ANSWER: 2.25

4. (a) HINT: The slope of the tangent line is given by $f'(4)$.
   ANSWER: $y = \frac{1}{4}x + 1$

(b) HINT: The area of the shaded region is

$$\int_0^4 (\frac{1}{4}x + 1) - \sqrt{x}\, dx.$$

ANSWER: $\frac{2}{3}$

5. (a) HINT: $f_x(x, y) = -10x - 5 + 10y$ and $f_y(x, y) = -6y + 4 + 10x$. Set both partial derivatives equal to 0 and solve the resulting system of equations.

   ANSWER: $(-0.25, 0.25)$

   (b) HINT: $A \approx f_y(4, 6)$ and $B \approx f_x(4, 6)$

   ANSWER: B is bigger

   (c) HINT: Each of the functions you are considering is a function of y. So, compute $f_y(1, 1)$, $f_y(3, 1)$, $f_y(6, 1)$, and $f_y(8, 1)$. These numbers are the slopes of the functions at the relevant point. Choose the largest.

   ANSWER: The function with the steepest graph at $y = 1$ is $f(8, y)$.

6. (a) HINT: Compute $f(2 + h) - f(2)$.

   ANSWER: $f(2 + h) - f(2) = 24h - 4h^2$

   (b) ANSWER: $f'(t) = -8t + 40$

   (c) ANSWER: $g(6) - g(2) = -10(2)(4) - 5(4)^2 + 60(4) = 80$

   (d) HINT: $\frac{g(t + h) - g(t)}{h} = -10t - 5h + 60$. Let h go to 0 to get the instantaneous speed.

   ANSWER: $g'(t) = 60 - 10t$

7. (a) ANSWER: $TR(q) = q^3 - 142.5q^2 + 5250q$

   (b) ANSWER: $TR'(q) = 0$ at $q = 25$ and $q = 70$. But 70 is not in the interval of consideration. So, the only critical number is $q = 25$. $TR''(25) < 0$ so TR has a local max at $q = 25$.

   (c) HINT: Compute $h(25)$.

   ANSWER: \$2312.50