NAME: ________________________________ QUIZ SECTION:______

Student ID #: ____________________________

Math 112 -- Winter 2016
Final Exam

HONOR STATEMENT:

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:________________________________

INSTRUCTIONS:

- When the exam starts, verify that your exam contains **9 pages** (including this cover page).
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless specifically instructed otherwise, you **must show all your work in order to get full credit**. The correct answer with incorrect or missing work may result in little or no credit.
- On problems in which you use a graph, show your work by clearly drawing & labeling any lines and points you use.
- If you use a guess-and-check method when an algebraic method is available, you will not receive full credit.
- You may round your final answers to two decimal digits. Don't round any values prior to the final answer.
- You are allowed to use a calculator, a ruler, and one sheet of notes. You have 2:50 hours for this exam.

GOOD LUCK!

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Total:	**100**	

$\boxed{1}$ (16 pts) Compute the indicated derivatives. DO NOT SIMPLIFY. Box your final answer.

a) $f(t) = \sqrt{\ln(t^2 - 3t) + 7}$

$f'(t) =$

b) $u = \dfrac{e^x \ln x}{x^2 + \frac{1}{x} - 7}$

$\dfrac{du}{dx} =$

c) $z = 2e^y x + \frac{y}{x} + \ln(xy^2) + x$

$\dfrac{\partial z}{\partial x} =$

$\dfrac{\partial z}{\partial y} =$

$\boxed{2}$ (6 pts) Suppose we do not have a formula for a certain function $f(x)$, but we know that:

$$f(m+h) - f(m) = \frac{12h}{(2+m+h)(5+m)}$$

Compute $f'(3)$. Show all steps clearly.

ANSWER: $f'(3) =$ ______________

$\boxed{3}$ (10 pts) Compute each of the following **integrals**. SIMPLIFY and $\boxed{\text{box}}$ your final answers.

a) $\displaystyle\int \frac{3}{x^2} - 2e^{2x} + \frac{7x^2+3}{x}\, dx$

b) $\displaystyle\int_9^{25} \frac{3}{\sqrt{t}} + 2\, dt$

$\boxed{\textbf{4}}$ (8 pts) The demand and supply functions for a product are:

$$\text{demand:} \quad p = \frac{77}{x+1}$$

$$\text{supply:} \quad p = 2 + 0.5x$$

where p is the price per unit, in dollars, and x is the number of units.

Compute the consumers surplus under pure competition.

ANSWER: Consumers Surplus = $ ____________________

(You may round your final answer to the nearest two decimal digits)

$\boxed{5}$ (12 pts) Two bicyclists, Anne and Bob, are next to each other at time $t = 0$, and travel along the same straight road. Their respective speeds at t hours are given by the functions:

Biker Anne's speed: $\boldsymbol{a(t) = 3t^2 - 10t + 16}$ miles/hour

Biker Bob's speed: $\boldsymbol{b(t) = 2t + 10}$ miles/hour

a) At what time during the first 1.5 hours are the two bikers farthest apart?

Answer: at $t =$___________ hours.

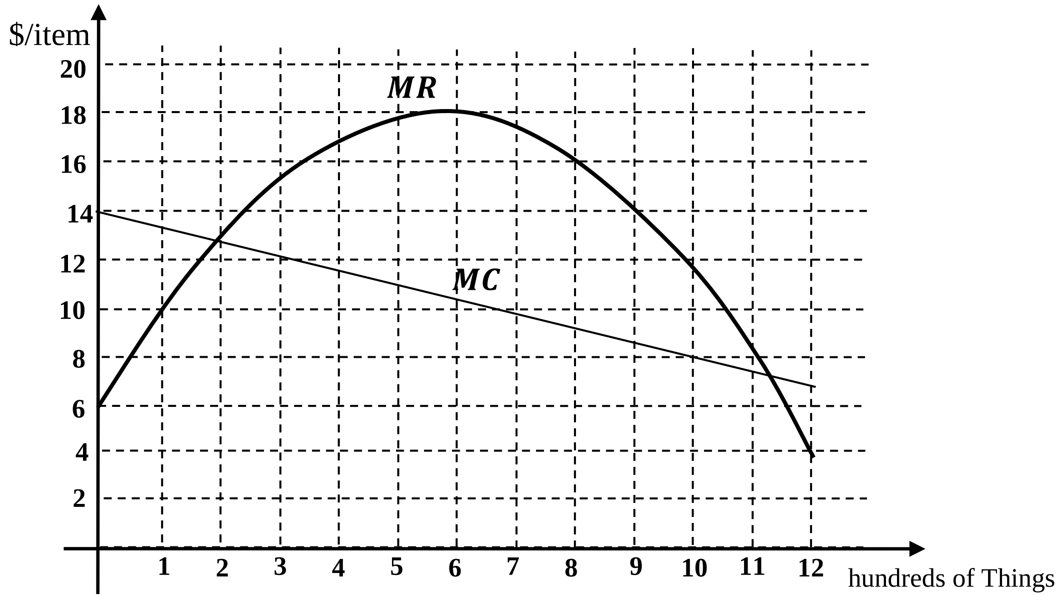
b) Which biker is ahead after 1 hour, and by how much? Show work.

Answer: Biker ________ is ahead by _________miles.

c) Recall that the instantaneous speed for Biker Bob is given by the linear function: $b(t) = 2t + 10$. Compute the **average speed** of Biker Bob over the time interval from $t = 1$ to $t = 2.5$ hours.

Answer: Bob's average speed was _________________ miles per hour.

$\boxed{6}$ (12 pts) The marginal revenue and marginal cost at q hundred Things are given by the graphs below.



You also know that your fixed costs are 2 hundred dollars.

a) Estimate your Total Cost for producing 300 Things. Show your work.

Answer: $TC(3) \approx$ ____________ hundred dollars

b) Estimate the minimal profit (maximal loss), and the quantity at which it occurs. Show work.

Answer: Min Profit / Max Loss $\approx$ _____________ hundred dollars, at $q \approx$ ______ hundred Things

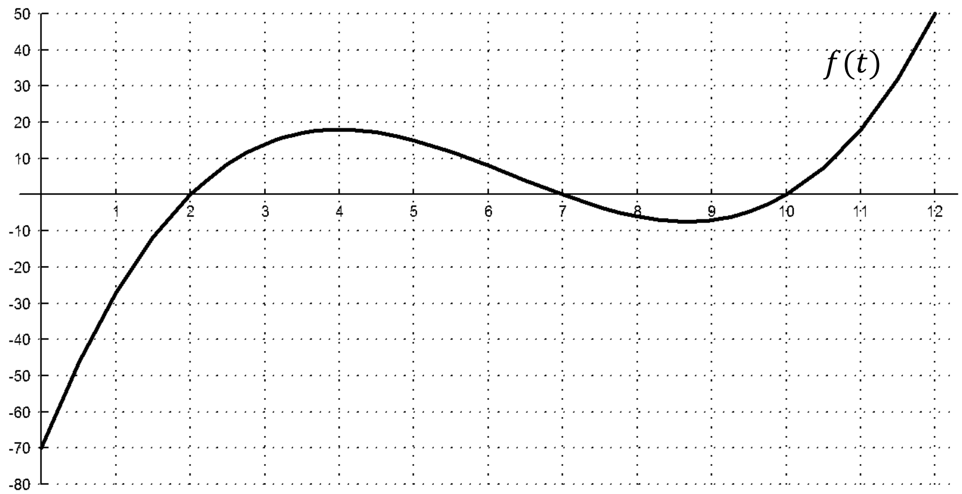
c) Estimate the change in revenue from $q = 3$ to $q = 4$ hundred Things. Show work.

Answer: _______________ hundred dollars

d) Does your profit increase or decrease if you produce and sell the 301st Thing? By approximately how much?

Answer: The profit increases/decreases (circle one) by about __________________ dollars

$\boxed{7}$ (10 pts) The following is the **graph of a function** $f(t)$.



Let $\boldsymbol{A(m) = \int_0^m f(t)\,dt}$ be the accumulated graph of $f(t)$. Answer the following questions. Read each question carefully!

a) For each part below, circle the correct answer. No need to justify.

i.	The value of $f(5)$ is	POSITIVE,	NEGATIVE,	or	ZERO.
ii.	The value of $f'(5)$ is	POSITIVE,	NEGATIVE,	or	ZERO
iii.	The value of $f''(5)$ is	POSITIVE,	NEGATIVE,	or	ZERO
iv.	The value of $A(7)$ is	POSITIVE,	NEGATIVE,	or	ZERO
v.	The value of $A'(7)$ is	POSITIVE,	NEGATIVE,	or	ZERO

b) Find the longest interval during which the derivative $f'(t)$ is **decreasing**.

Answer: from $t =$_____ to $t =$_____

c) Estimate $\boldsymbol{A'(9)}$.

Answer: $A'(9) \approx$_________________

d) $f(t)$ has inflection points at $x =$ _________________(*list all, no need to justify*)

e) The local minima of $\boldsymbol{A(m)}$ are at $m =$_____________________________(*list all, no need to justify*)

$\boxed{8}$ (14 Points) You produce and sell flat-screen TV's and Blu-ray Players.

(a) (2 pts) Suppose you sell each TV for \$2000 and each Player for \$500. Give a formula for the total revenue $R(x, y)$, in dollars, which results from selling x TV's and y Players.

ANSWER: $R(x, y) =$ ______________________

(b) Suppose your profit from selling x TV's and y Players is given by the function:

$$\boldsymbol{P(x, y) = 0.1x^2 + 0.1\,y^2 - 0.6xy + 300x + 100y - 1000}$$

i. (2 pts) Compute the two partial derivatives of your profit function.

$P_x(x, y) =$ ______________________

$P_y(x, y) =$ ______________________

ii. (6 pts) Find all candidates (x, y) for local minima or maxima of the profit $P(x, y)$.

Answer: $(x, y) =$ ______________________

iii. (4 pts) Suppose you've produced and sold 300 TV's and 250 Players. Use a partial derivative to estimate the increase in your profit if you sell one more TV. Show your work, clearly.

Answer: Profit will change by about \$ ______________

$\boxed{9}$ (12 pts) The Demand Curve for selling Items has the formula:

$$p = 1 - 0.2\sqrt{q},$$

where the quantity q is in hundreds of Items and the price p is in dollars per Item.

The total cost (in hundreds of dollars) to produce q hundred Items is given by the formula:

$$TC(q) = 0.01q + 0.5.$$

Let $\boldsymbol{P(q)}$ denote the **profit** (in hundreds of dollars) you earn by producing and selling q hundred Items.

a) Determine the formula for the **profit** $P(q)$, as an expression in q. Simplify your answer.

ANSWER: $P(q) =$ ____________________________________

b) Compute the critical number(s) of the profit.

ANSWER: $q =$ ________________ hundred Items

c) Use the Second Derivative Test to determine whether each critical number you found above gives a local maximum or a local minimum for the profit function, $P(q)$. Show work clearly, and box your answer(s).