Solutions to Math 112 W'15 MT1

1. (10 points) You do not have to simplify your answers but make sure they are clearly written with properly used parentheses where necessary.

   (a) Find $f'(x)$ if $f(x) = \dfrac{(5x-7)^2}{x^3+5}$.

$$f'(x) = \frac{2(5x-7)\cdot 5\cdot (x^3+5) - (5x-7)^2\cdot 3x^2}{(x^3+5)^2}$$

   (b) Compute $\dfrac{d}{dx}\left((3x^3-5x+8)(2x^{3/2}+1)\right)$

$$= (9x^2-5)(2x^{3/2}+1) + (3x^3-5x+8)(3x^{1/2})$$

   (c) Find the second derivative y'' if $y = 0.25x^4 - \sqrt{x} + \dfrac{2}{x^3}$ $= 0.25x^4 - x^{1/2} + 2x^{-3}$

$$y' = x^3 - \frac{1}{2}x^{-1/2} - 6x^{-4}$$

$$y'' = 3x^2 + \frac{1}{4}x^{-3/2} + 24x^{-5}$$

2. (14 points) Two toy cars, a Gray one and a Black one, are on a two lane straight track. At $t = 0$, they are both at the point of reference shown by the vertical line. The distance is measured positive to the right. The **distance** function of the Gray Car is

$$\mathbf{G}(t) = -1.25t^2 + 20t$$

where **G** is in centimeters and the time t is in seconds. The **velocity** of the Black Car is

$$\mathbf{b}(t) = -0.25t^2 + 5t - 16$$

which is in centimeters per second. Everything happens in $0 \leq t \leq 12$. Answer the following questions.

(a) The following are two snapshots. Put arrows to show which direction each car was traveling when the picture was taken. Show your computations under the pictures. The Gray Car is in the top lane, in case the colors came too close in your copy.

t=1   t=5

$G'(t) = -2.5t + 20$

$G'(1) = 17.5 > 0$

$b(1) = -0.25 + 5 - 16 < 0$

$G'(5) = -2.5(5) + 20 = -12.5 + 20 > 0$

$b(5) = -0.25(5)^2 + 5^2 - 16$

$= -6.25 + 25 - 16 > 0$

(b) At what time is the Gray Car farthest to the right?

when $G'(t) = 0$ so $-2.5t + 20 = 0$

$t = \frac{20}{2.5} = 8$ seconds

(c) At what time is the Black Car momentarily at rest?

when $b(t) = 0$

$-0.25t^2 + 5t - 16 = 0$

$t = \frac{-5 \pm \sqrt{25 - 4(-0.25)(-16)}}{2(-0.25)} = \frac{-5 \pm 3}{-0.5} = 10 \pm 6$

$= \boxed{4}$ or 16

$0 \leq t \leq 12$ in the problem.

The **distance** function of the Gray Car is $\mathbf{G}(t) = -1.25t^2 + 20t$ and the **velocity** of the Black Car is $\mathbf{b}(t) = -0.25t^2 + 5t - 16$.

(d) At what time is the distance between the two cars greatest?

when $G'(t) = b(t)$

$$-2.5t + 20 = -0.25t^2 + 5t - 16$$

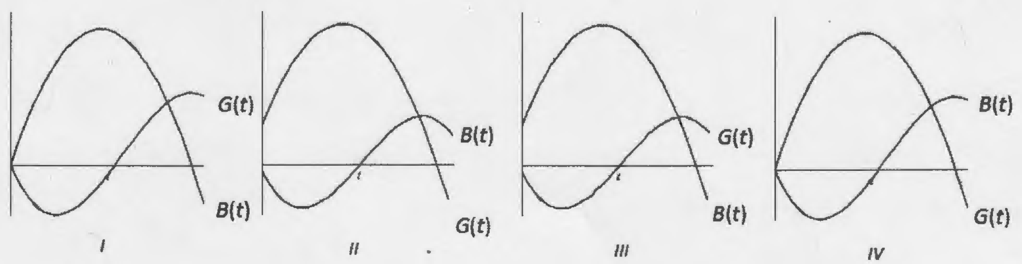
$$0.25t^2 - 7.5t + 36 = 0$$

$$t = \frac{7.5 \pm \sqrt{(7.5)^2 - 4(0.25)(36)}}{2(0.25)} = \frac{7.5 \pm 4.5}{0.5}$$

$$= 15 \pm 9 \quad \text{so} \quad \boxed{t = 6}$$

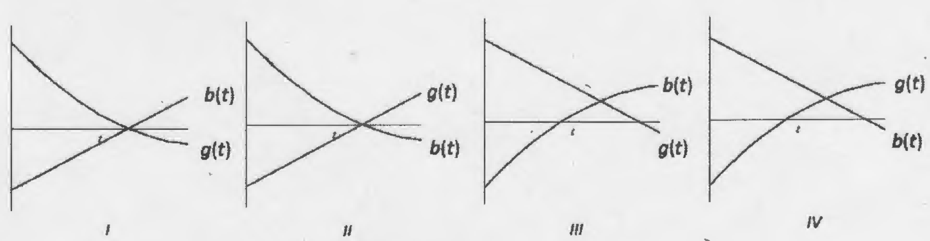
(e) The following are possible graphs for the **distance** functions $\mathbf{G}(t)$ and $\mathbf{B}(t)$. Circle the correct one.

$G(0) = B(0)$
so
I or IV
$G'(0) = 20 > 0$
so IV



(f) The following are possible graphs for the **velocity** functions $\mathbf{g}(t)$ and $\mathbf{b}(t)$. Circle the correct one.

$g(0) = G'(0) = 20 > 0$
so I or III
$g(t)$ is linear
So III



3. (14 points) The Total Cost and Total Revenue for producing and selling Bonjouks are given by

$$\mathbf{TC}(q) = 0.013q^3 - 1.7q^2 + 73.4q + 650 \qquad \mathbf{TR}(q) = -q^2 + 140q$$

where q is the number of boxes of Bonjouks and the Total Revuenue and Total Cost are in dollars.

(a) What is the maximum profit?

max profit when $P'(q)=0$

$P'(q) = TR'(q) - TC'(q) = (-2q+140) - (0.039q^2 - 3.4q + 73.4)$

$= -0.039q^2 + 1.4q + 66.6 = 0$

$q \doteq \dfrac{-1.4 \pm \sqrt{(1.4)^2 - 4(-0.039)(66.6)}}{2(-0.039)} \approx \dfrac{-1.4 \pm 3.5142}{-0.078} \approx 63$ boxes (other $q<0$)

$P(63) = (-63^2 + 140 \cdot 63) - (0.013 \cdot 63^3 - 1.7 \cdot 63^2 + 73.4 \cdot 63 + 650)$

$= 4851 - 1777.51 = 3073.49$ dollars

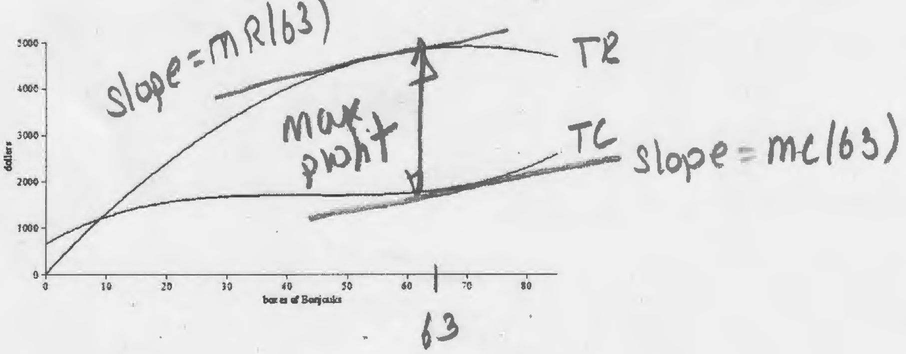
(b) Give the longest interval where Total Revenue and Marginal Cost are both increasing.

TR ↗ when $TR' = MR > 0$ $\quad -2q + 140 > 0$ so $q < 70$

MC ↗ when $MC' > 0$ so $0.078q - 3.4 > 0$ so $q > 43.59$

interval: $43.59 < q < 70$

(c) Below are graphs of Total Cost and Total Revenue. Mark the graphs as TC and TR. Show Maximum Profit, Marginal Revenue and Marginal Cost on the graph at the quantity where profit is maximized (from your answer in part (a)).



4. (12 points) A function f has the property that

$$f(A+B) - f(A) = 20AB + 10B^2 - 7B.$$

(a) If $f(0) = -12$, what is $f(2)$?

$f(0+2) - f(0) = 20\cdot0\cdot2 + 10\cdot2^2 - 7\cdot2 = 0 + 40 - 14 = 26$

So $f(2) - (-12) = 26$

$f(2) = 14$

(b) What is $f'(x)$?

$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} \frac{20xh + 10h^2 - 7h}{h}$

$= \lim_{h\to0} 20x + 10h - 7 = 20x - 7$

(c) Which one is more: The average rate of change of f from $x = 1$ to $x = 5$ or the instantaneous rate of change of f at $x = 2$?

Ave. rate of change from $x=1$ to $x=5$ $= \frac{f(5)-f(1)}{5-1} = \frac{f(1+4)-f(1)}{4} = \frac{20\cdot1\cdot4 + 10\cdot4^2 - 7\cdot4}{4} = 53$

Instanteneous rate of change at $x=2$ $= f'(2) = 20(2) - 7 = 33$

So ave. rate of change is more.

(d) For which value(s) of x is the tangent line to the graph of $y = f(x)$ horizontal?

$f'(x) = 20x - 7 = 0$

$x = \frac{7}{20}$