$\qquad$
STUDENT ID: $\qquad$

## Math 112 Midterm 1

Winter 2016

## HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

## SIGNATURE:

$\qquad$

- Do not open the test until instructed to do so. Please turn your cell phone OFF now.
- This exam consists of this cover sheet followed by five problems on five pages.

When the test starts, check that you have a complete exam.

- This exam is closed book. You may use one double-sided, handwritten $8 \frac{1}{2} \times 11$ page of notes, a ruler, and a non-graphing calculator. Put everything else away. You may not share notes.
- Unless otherwise indicated, you must show your work and justify your answers. The correct answer with incomplete or missing supporting work may result in no credit.
- Place your final answer in the indicated spaces. Unless otherwise specified, you may round your final answer to two digits after the decimal. Do NOT round off before your final answer.
- Read each question carefully first, and think about how to proceed. In order not to run out of time, try to not spend more than about 10 minutes per page.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so. Raise your hand if you have a question. GOOD LUCK!

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 8 |  |
| 3 | 13 |  |
| 4 | 6 |  |
| 5 | 13 |  |
| Total | 50 |  |

1. (10 points) Compute the indicated derivatives. Do NOT simplify your answers, but BOX your final answers.
(a) If $f(x)=\left[x^{3}\left(x^{2}+3 x\right)\right]^{4}$, compute $f^{\prime}(x)$.
(b) If $y=\frac{5}{2 x^{3}+7}$, compute $\frac{d^{2} y}{d x^{2}}$.
2. (8 points) The following is the graph of a function $y=f(x)$.


Use this graph to answer the following questions.
DRAW and LABEL on the graph any lines you use, and be as precise as possible.
(a) Estimate the value of $\frac{f(5.001)-f(5)}{0.001}$.

$$
\text { ANSWER: } \frac{f(5.001)-f(5)}{0.001} \approx
$$

$\qquad$
(b) Find all values of $x$ where $f^{\prime}(x)=20$.

ANSWER: At $x=$ $\qquad$
(c) Find a positive value $h$ such that $\frac{f(3+h)-f(3)}{h}=0$.

ANSWER: $h=$ $\qquad$
(d) Sketch the graph of $f^{\prime}(x)$. I'm only looking for rough shape and correct $x$-intercepts.

3. (13 points)
(a) (8 pts) A car drives on a straight road. Its distance from a certain point is given by a function $D(t)$, where the time $t$ is in seconds and the distance $D(t)$ is in feet.
We don't have the formula for $D(t)$, but we know that from time $t=1$ to time $t=1+h$ seconds, the change in distance for this car is given by the formula:

$$
D(1+h)-D(1)=\left(h^{2}+3 h\right) \sqrt{1+h}
$$

i. Compute the car's average speed over the interval from $t=1$ to $t=4$ seconds.

ANSWER: $\qquad$ feet per second
ii. Compute the car's instantaneous speed at $t=1$ seconds. Show all steps.

ANSWER: $\qquad$ feet per second
(b) (5 pts) Suppose $d(t)=t \sqrt{t^{2}+7}$ is the distance an object traveled, in meters, after $t$ minutes. Compute this object's instantaneous speed at $t=3$ minutes.
$\qquad$ meters per minute
4. (6 points) A Red balloon and a Green balloon rise and fall. When we start watching, at $t=0$, both balloons are 50 feet above the ground. Their altitudes at time $t$ minutes are given by functions $R(t)$ and $G(t)$, both measured in feet. The graphs below show the instantaneous rates of ascent of the balloons. That is, the graphs show $R^{\prime}(t)$ and $G^{\prime}(t)$, in feet per minute.


In this problem you do not have to show work or explain your answers.
(a) For each of the following statements circle the correct answer: True (T), False (F), or cannot tell based on the given information (CT).

At $t=2 \mathrm{~min}$, Green balloon's altitude is higher than 50 feet. $\mathrm{T} \quad \mathrm{F} \quad \mathrm{CT}$

At $t=2$ min, the Green balloon is higher than the Red one. $\mathrm{T} \quad \mathrm{F}$ CT

The distance between the balloons is greater at $t=3$ than at $t=2 \quad \mathrm{~T} \quad \mathrm{~F} \quad \mathrm{CT}$

At $t=10 \mathrm{~min}$, the Green balloon is rising faster than the Red one. $\mathrm{T} \quad \mathrm{F}$ CT

At $t=10 \mathrm{~min}$, Green balloon's acceleration is greater than the Red's. $\mathrm{T} \quad \mathrm{F}$ CT
(b) Find the longest time interval during which the Green balloon is descending.
$\qquad$ to $t=$ $\qquad$ minutes
5. (13 points) You sell Items. The formulas for the total revenue and total cost, in hundreds of dollars, for selling and producing $q$ hundred Items are:
total revenue: $T R(q)=30 q \quad$ total cost: $T C(q)=q^{3}-15 q^{2}+78 q+10$
(a) What is the marginal cost at 2 hundred items? Include correct units, and interpretation.

ANSWER: The marginal cost at 200 items is: $\qquad$ Units: $\qquad$

Interpretation/meaning:
(b) Find the longest interval on which marginal revenue exceeds marginal cost.
(HINT: Sketch the graphs of MR and MC on the same set of axes.)

ANSWER: From $q=$ $\qquad$ to $q=$ $\qquad$ hundred Items
(c) What is the maximum value of profit? (Include units)

ANSWER: Maximum profit is: $\qquad$ Units: $\qquad$

