Solutions to w'o5, m112, MTII

1. (13 points)
(a)

$$
\begin{aligned}
\frac{d}{d x} \sqrt{3+5 \ln x} & =\frac{d}{d x}(3+5 \ln \dot{x})^{\frac{1}{2}} \\
& =\frac{1}{2}(3+5 \ln x)^{-1 / 2}\left(0+\frac{5}{x}\right) \\
& =\frac{5}{2 x \sqrt{3+5 \ln x}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\int \frac{2 \sqrt{x}+7 x^{2}+9}{5 x^{3}} d x & =\int \frac{2}{5} x^{-5 / 2}+\frac{7}{5} \frac{1}{x}+\frac{9}{5} x^{-3} d x \\
& =\frac{2}{5} \frac{x^{-3 / 2}}{-3 / 2}+\frac{7}{5} \ln |x|+\frac{9}{5} \frac{x^{-2}}{-2}+C \\
& =\frac{-4}{5} x^{-3 / 2}+\frac{7}{5} \ln |x|-\frac{9}{10 x^{2}}+C
\end{aligned}
$$

(c) $\int \sqrt{3+7 x} d x$

Hint: Check your answer by differentiating.

$$
\begin{aligned}
=\int(3+7 x)^{1 / 2} d x & =\frac{1}{7} \frac{(3+7 x)^{3 / 2}}{3 / 2}+C \\
& =\frac{2}{21}(3+7 x)^{3 / 2}+C
\end{aligned}
$$

2. (19 points) You produce and sell Palabras. Your Marginal Revenue and Marginal Cost (both in dollars per Palabra) are given by the formulas

$$
\mathbf{M R}(q)=20-0.65 q \quad \text { and } \quad \mathbf{M C}(q)=0.15 q^{2}-2.4 q+10.8
$$

where $q$ is in thousands of Palabras.
(a) At what quantity is the profit maximized? Give your answer to the nearest Palabra.

$$
\begin{aligned}
& 20-0.65 q=0.15 q^{2}-2.49+10.8 \\
& 0=0.15 q^{2}-1.75 q-9.2 \\
& q=\frac{1.75 \pm \sqrt{(1.75)^{2}-4(0.15)(-9.2)}}{0.3}=15.599 \text { thousand } \\
& \text { or } 15,599 \text { Palabrus }
\end{aligned}
$$

$(M R \mid 0)=20>M C(0)=10.5$ so $M R>M C$ when $y<15.599$ (b) Give the formal $m R(16)<m C(16))$

$$
\begin{aligned}
& \text { jive the ernumas for Variable Cost and Total Revenue. } \\
& T R=\int 20-0.65 q \frac{d q}{}=20 q-0.32 q^{2}+C
\end{aligned}
$$

$\sin \left(e \operatorname{TR}(0)=0, \quad T R=20 q-0.325 q^{2}\right.$

$$
\begin{aligned}
& \operatorname{since} T R(0)=0, \quad T R=20 q-0.325 q^{2} \\
& V C=\int 0.15 q^{2}-2.4 q+10.8=0.05 q^{3}-1.2 q^{2}+10.8 q+C \\
& \text { sine } V C(0)=0 \quad, V C=0.05 q^{3}-1.2 q^{2}+10.8 q
\end{aligned}
$$

(c) If you sell $\mathrm{q}=10$ thousand Palabras, then your profit is 123 thousand dollars. Find the value of your fixed costs. Give units with your answer.

$$
\begin{aligned}
& \text { so } \begin{aligned}
F C & =T R 110)-V C(10)-123 \\
& =(200-32.5)-(50-120+108)-123 \\
& =6.5 \text { thousand dollars }
\end{aligned}
\end{aligned}
$$

(d) Find the maximum profit. Give units with your answer.

$$
\begin{aligned}
& \text { Find the maximum profit. Give units with your answer. }(15.599)-6.5 \\
& \begin{aligned}
P(15.599) & =T R(15.599)-V C(259-6.5 \\
& =232.898-66.259-69 \\
& =160.139
\end{aligned}
\end{aligned}
$$

(e) There are four identical graphs of Marginal Revenue and Marginal Cost below. Show the quantity which is described below each on the graph (as points, horizontal or vertical coordinates, slopes of lines or areas, whichever applies). Do not try to compute or estimate the quantities.


The maximum revenue


The rate of change of Marginal Cost at $q=15$.


The Variable Cost at $q=4$


The change in profit from $q=2$ to $q=8$.
3. (10 points) A purple balloon moves up and down and its rate of change of altitude is given by

$$
p(t)=2 t^{2}-15 t+18
$$

where $t$ is in minutes and the rate is in feet per minute. Below is the graph of $p(t)$ to help you visualize.
(a) In which intervals) is the altitude increasing?

$$
\begin{aligned}
& \text { which interval(s) is the altitude increasing? } \\
& \begin{array}{c}
2 t^{2}-15 t+18=(2 t-3)(t-6) \\
t=3 / 2 \quad t=6
\end{array}
\end{aligned}
$$

allikele increasing when $p(t)>0$ : $0<t<\frac{3}{2}$ or $t>6$.

(b) If we graph the altitude function of the balloon, in which intervals) will it be concave down? when $p(t)$ is increasing
$4 t-15=0$ when $t=\frac{15}{4}$ so when $t>\frac{15}{4}$.
(c) What is the average rate of change $8^{\frac{1}{2} \text { altitude of the balloon during the firstiminutes }}$

$$
\begin{aligned}
\frac{P(2)-P(0)}{2-0} & =\frac{1}{2} \int_{0}^{2} 2 t^{2}-15 t+18 d t \\
& =\frac{1}{2}\left(\frac{2 t^{3}}{3}-\frac{15 t^{2}}{2}+\left.18 t\right|_{0} ^{2}\right)=\frac{1}{2}\left(\frac{16}{3}-30+36\right) \\
& \left.=\frac{34}{6}=\frac{17}{3} \mathrm{ft}\right)_{\text {min. }} .
\end{aligned}
$$

(d) If the balloon is at feet at $t=3$, does it ever touch the ground? This should not involve solving a cubic equation.
(only) Local min at $t=b_{6}$

$$
\begin{aligned}
P(6)-P(3) & =\int_{3}^{3} 2 t^{2}-15 t+18 d t \\
& =\frac{2 t^{3}}{2}-\frac{15 t^{2}}{2}+\left.18 t\right|_{3} ^{6} \\
& =\frac{2}{3}\left(6^{3}-3^{3}\right)-\frac{15}{2}\left(6^{2}-3^{2}\right)+18(6-3) \\
& =2 \cdot 3^{2} \cdot 7-\frac{15}{2} \cdot 3^{3^{2}} \cdot 3+18 \cdot 3=9\left(14-\frac{45}{2}+6\right)=-\frac{45}{2}
\end{aligned}
$$

so $P(6)=23-22.5=0.5$, 5 no it does not.
4. (8 points) Below is the graph of the demand and supply functions for a product. The formula for the demand function is $p=\frac{630}{x+5}$ and the supply function is $p=2 x+27$ Mark the functions as DEMAND and SUPPLY on the graph and find the consumer's surplus which is represented by the shaded area.

equilibnum: $\quad \frac{630}{x+5}=2 x+27$

$$
630=(2 x+27)(x+5)
$$

$$
=2 x^{2}+37 x+135
$$

$$
\begin{aligned}
0 & =2 x^{2}+37 x-495 \\
x & =\frac{-37 \pm \sqrt{37^{2}-4 \cdot 2(-495)}}{4}=9 \quad p=2.9+27=45 \\
C S=\int_{0}^{9} \frac{630}{x+5} d x-9.45 & =\left.630 \ln (x+5)\right|_{0} ^{9}-405 \\
& =630 \ln \left(\frac{14}{5}\right)-405 \\
& \simeq 243.66
\end{aligned}
$$

