1. (10 points)

(a) (6 points) Differentiate the following function. BOX your final answer. No need to simplify.

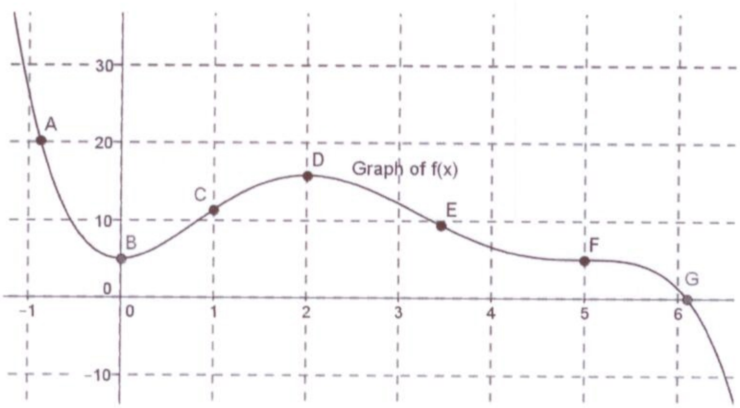
$$y = \frac{\ln(x^2 - 3x)}{e^{5x} + e^2}$$

$$\frac{dy}{dx} = \frac{[\ln(x^2-3x)]'(e^{5x}+e^2) - \ln(x^2-3x)[e^{5x}+e^2]'}{(e^{5x}+e^2)^2}$$

(Quotient Rule first)

$$= \boxed{\frac{\left(\frac{1}{x^2-3x}\cdot(2x-3)\right)(e^{5x}+e^2) - \ln(x^2-3x)(e^{5x}\cdot 5)}{(e^{5x}+e^2)^2}}$$

(b) (4 points) Based on the graph of $f(x)$ shown below, identify which of the points A through G marked on the graph satisfy each of the following conditions.



(List all points that apply, no need to justify your answers)

i. Critical point(s) for $f(x)$: B, D, F

ii. Inflection point(s) for $f(x)$: C, E, F

iii. Point(s) where $f'(x) = 0$ and $f''(x) = 0$: F

iv. Point(s) where $f'(x) > 0$ and $f''(x) = 0$: C

2. (14 points) The total revenue, in thousands of dollars, for selling q thousand Items is given by:

$$TR(q) = \frac{1}{6}q^4 - \frac{16}{3}q^3 + 48q^2 + 100q.$$

(a) Compute the following derivatives:

$MR \longrightarrow TR'(q) = \frac{1}{6}(4q^3) - \frac{16}{3}(3q^2) + 48(2q) + 100 = \frac{2}{3}q^3 - 16q^2 + 96q + 100$

$MR' \longrightarrow TR''(q) = 2q^2 - 32q + 96$

$MR'' \longrightarrow TR'''(q) = 4q - 32$

(b) Find all the critical values of the **marginal revenue** function.

$MR'(q) = 2q^2 - 32q + 96 = 0$

$q^2 - 16q + 48 = 0$

Q.F. or factor : $q = 4,\ q = 12$

ANSWER: (list all) $q =$ 4, 12 thousand Items.

(c) At each of the quantities you found in part (a), use either the First or the Second Derivative Test to determine whether the **graph of marginal revenue** has a local maximum, a local minimum, or neither. Make sure to specify which test you use and show your work.

Second Derivative Test:

$MR''(4) = 4(4) - 32 < 0 \Rightarrow$ local max at $q = 4$

$MR''(12) = 4(12) - 32 > 0 \Rightarrow$ local min at $q = 12$

OR

First Derivative Test

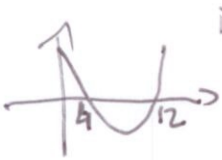
q		4		12	
$MR' = 2q^2 - 32q + 96$	+ + + +	0	− −	0	+ + + +
MR	↗	local max	↘	local min	↗

ANSWER: Local MAX at $q =$ 4, Local min at $q =$ 12

(d) Find the longest interval on which the **total revenue** function $TR(q)$ is **concave down**. Justify.

TR is concave down when $TR'' < 0$

i.e. $2q^2 - 32q + 96 < 0$



← This happens between its roots

ANSWER: From $q =$ 4 to $q =$ 12 thousand Items

(e) Does the **total revenue** function have a local maximum at $q = 10$ thousand Items? Justify.

$TR'(10) = \frac{2}{3}(1000) - 16(100) + 96(10) + 100 \neq 0$

ANSWER: Yes/No because: $q = 10$ is not a C.V. for TR

3. (9 points) Evaluate the following integrals. Simplify and BOX your final answer.

(a) $\int \left[\left(1+x^2\right)\left(\frac{1}{x}+2x\right) + e^{0.5x} \right] dx =$

$$= \int \frac{1}{x} + \underbrace{2x + x}_{} + 2x^3 + e^{0.5x}\, dx$$

$$= \int \frac{1}{x} + 3x + 2x^3 + e^{0.5x}\, dx$$

$$= \ln(x) + \frac{3}{2}x^2 + \frac{2}{4}x^4 + \frac{1}{0.5}e^{0.5x} + C$$

$$= \boxed{\ln(x) + \frac{3}{2}x^2 + \frac{1}{2}x^4 + 2e^{0.5x} + C}$$

(b) $\int_1^4 \left(9x^2 - \frac{2}{\sqrt{x}}\right) dx = \int_1^4 9x^2 - 2x^{-1/2}\, dx$

$$= \left(9\,\frac{1}{3}x^3 - \frac{2}{(1/2)}x^{1/2}\right)\Big|_1^4$$

$$= \left(3x^3 - 4\sqrt{x}\right)\Big|_1^4$$

$$= \left(3(64) - 4\sqrt{4}\right) - \left(3(1) - 4\sqrt{1}\right)$$

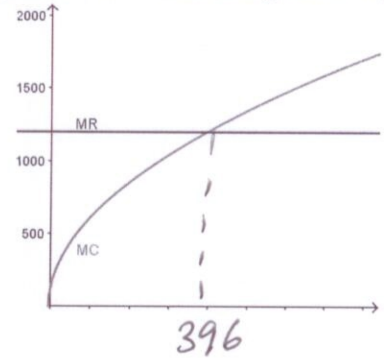
$$= 184 \qquad - (-1)$$

$$= \boxed{185}$$

4. (9 points) A company is selling Things. Suppose that the marginal revenue and marginal cost, in dollars per Thing, at q Things are:

$$MR(q) = 1200$$

$$MC(q) = 60\sqrt{q+4}$$



(a) What number of Things results in a maximal profit?

MR = MC: $1200 = 60\sqrt{q+4}$

$\sqrt{q+4} = \frac{1200}{60} = 20$

Squaring: $q+4 = 400$

$q = 396$

[Picture: MR > MC before $q = 396$ ⟹ Profit ↑ } max at 396
MR < MC after ⟹ Profit ↓]

ANSWER: $q =$ 396 Things

(b) What is the maximum profit, assuming that the fixed costs are \$1000? Show all work.

$TR(q) = 1200q + C$, $TR(0) = 0 \Rightarrow 1200(0) + C = 0 \Rightarrow C = 0$

$\boxed{TR(q) = 1200q}$

$TC(q) = 60 \frac{1}{\left(\frac{3}{2}\right)}(q+4)^{3/2} + C = 40\left(\sqrt{q+4}\right)^3 + C$

$TC(0) = FC = 1000 \Rightarrow 40\left(\sqrt{0+4}\right)^3 + C = 1000$

$40(8) + C = 1000$

$C = 1000 - 320 = 680$

$\boxed{TC(q) = 40\left(\sqrt{q+4}\right)^3 + 680}$

Profit: $\boxed{P(q) = (1200q) - \left(40\left(\sqrt{q+4}\right)^3 + 680\right)}$

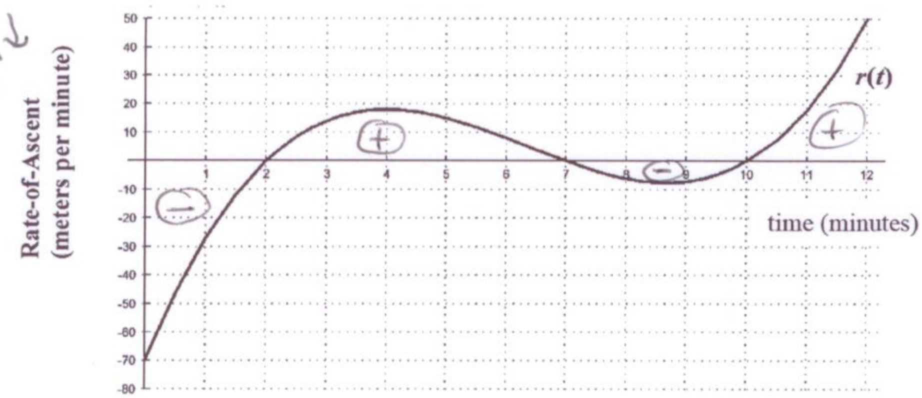
At $q = 396$: $1200(396) - \left(40\left(\sqrt{400}\right)^3 + 680\right)$

$475{,}200 - 320{,}680$

ANSWER: 154,520 dollars.

5. (8 points) The graph below respresents the RATE of ASCENT $r(t)$ for a balloon. In this problem, you need not show work.

⊖ = descending ↓
⊕ = ascending ↑



(a) List all times at which the **altitude graph** of this balloon has horizontal tangents:

$A'(t) = r(t) = 0$

$t =$ 2, 7, 10

(b) At what time is the balloon at its lowest altitude? At $t =$ 2

(c) Give the longest time interval over which the balloon is rising and getting slower.

rising when $r(t)$ above x-axis
getting slower when $r(t)$ ↓

From $t =$ 4 to $t =$ 7

(d) Define a new function $A(m) = \int_0^m r(t)\,dt$, where $r(t)$ is the function in the graph above.

i. Estimate $A(2) \simeq$ -70 ← $\int_0^2 r(t)\,dt = -$(area above $r(t)$ & below x-axis from $t=0$ to $t=2$)

ii. At what value(s) of m does $A(m)$ have a local minimum? $m =$ 2, 10

iii. Which of the following does the function $A(m)$ represent? Circle all the correct answers.

The altitude of the balloon at time $t = m$.

The change in altitude of the balloon from its initial position.

The velocity of the balloon at time $t = m$.

The change in the velocity of the balloon from its initial velocity

An antiderivative of $r(t)$

A derivative of $r(t)$