Math 120B **Autumn 1999**

Midterm 1 Solutions

1. (a) We need to solve

$$-\frac{1}{2}x^2 + \frac{3}{10}x + \frac{2}{5} = 0.06 = \frac{3}{50}.$$

Clearing denominators gives the equivalent equation

$$25x^2 - 15x - 17 = 0.$$

The desired solution is

$$\frac{15 - 5\sqrt{77}}{50} \approx -58 \text{ ft}$$

(b)
$$\begin{aligned} h(x) &= -\frac{1}{2}x^2 + \frac{3}{10}x + \frac{2}{5} - x(x-3) \\ &= -\frac{3}{2}x^2 + \frac{33}{10}x + \frac{2}{5} \end{aligned}$$

(c) The vertex form for $h(x)$ is obtained as follows:

$$\begin{aligned} h(x) &= -\frac{3}{2}\left(x^2 - \frac{11}{5}x - \frac{4}{15}\right) \\ &= -\frac{3}{2}\left\{\left(x - \frac{11}{10}\right)^2 - \frac{121}{100} - \frac{4}{15}\right\} \\ &= -\frac{3}{2}\left\{\left(x - \frac{11}{10}\right)^2 - \frac{443}{300}\right\} \\ &= -\frac{3}{2}\left(x - \frac{11}{10}\right)^2 + \frac{443}{200} \end{aligned}$$

Therefore the maximum height is

$$\frac{443}{200} \text{ hundred ft } \approx 222 \text{ ft.}$$

(d) Solve $-\frac{1}{2}x^2 + \frac{3}{10}x + \frac{2}{5} = x^2 - 3x$, or equivalently

$$15x^2 - 33x - 4 = 0.$$

The sensible solution is

$$\begin{aligned} x &= \frac{33 + \sqrt{1329}}{30} \\ &\approx 2.32 \text{ hundred ft} \\ &\approx 232 \text{ ft.} \end{aligned}$$

Now plug this into either function to get the y-coordinate.

2. (a) The positive solution of $2500 - 16t^2 = 0$ is $t = \dfrac{50}{4} = 12.5$ sec.

(b) $h(t) = 2500 - 16t^2 + 100 = 2600 - 16t^2$. This is valid for $0 \le t \le 12.5$.

(c) The time since 12.5 seconds is $t - 12.5 : h(t) = -\frac{1}{2}(t - 12.5)$.

(d) $-\frac{1}{2}(t - 12.5) = -100 \Rightarrow t = 106.25$ sec.

(e) $h(t) = \begin{cases} 2600 - 16t^2 & 0 \le t \le 12.5 \\ -\frac{1}{2}(t - 12.5) & 12.5 \le t \le 106.25 \end{cases}$

3. (a) $h(t) = \frac{5}{4}t$, $h(t) = 12 - \sqrt{49 - (t-4)^2}$, $h(t) = \sqrt{144 - (t-11)^2}$

(b) $h(t) = \begin{cases} \frac{5}{4}t \text{ when} & 0 \le t \le 4 \\ 12 - \sqrt{49 - (t-4)^2}, & 4 \le t \le 11 \\ \sqrt{144 - (t-11)^2}, & 11 \le t \le 23 \end{cases}$

(c)
$$\begin{aligned} 12 - \sqrt{49 - (t-4)^2} = 8 &\Rightarrow t = 4 \pm \sqrt{33} \\ &\Rightarrow t = 4 + \sqrt{44} \\ \sqrt{144 - (t-11)^2} = 8 &\Rightarrow t = 11 \pm \sqrt{80} \\ &\Rightarrow t = 11 + \sqrt{80}. \end{aligned}$$

The time spent above 800 ft is $11 + \sqrt{80} - (4 + \sqrt{33}) = 7 + \sqrt{80} - \sqrt{33} \approx 10.2$ min.

4. (a) $g(f(x)) = \sqrt{1 - (2x-3)^2}$

(b)
$$\begin{aligned} 1 - (2x-3)^2 \ge 0 &\Rightarrow |2x - 3| \le 1 \\ &\Rightarrow -1 \le 2x - 3 \le 1 \\ &\Rightarrow 1 \le x \le 2 \end{aligned}$$

(c)
$$\begin{aligned} h(3+z) - H(3) &= 2(3+2)^2 - (3+2) - 15 \\ &= 2z^2 + 11z \\ &= z(2z + 11) \\ \frac{h(3+z) - h(3)}{z} &= 2z + 11 \end{aligned}$$