Name: _____ **Student Number:** _____

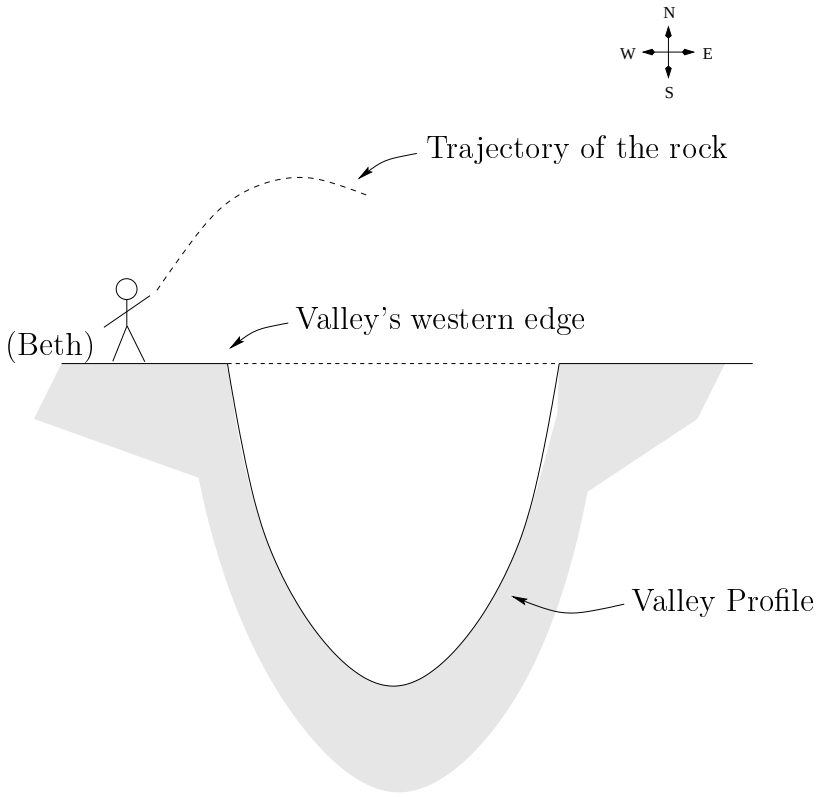
Math 120B **Autumn 1999**

Midterm 1

Instructions

- You will have 50 minutes for this exam.
- The exam is closed book, but you are allowed one $8\frac{1}{2}'' \times 11''$ page of notes (both sides) in your own handwriting.
- You **must show your** work in order to receive credit.
- Please keep the following in mind. You are being tested on *your* knowledge and proficiency over the topics covered thus far in this course. So, for example, if you need to solve an equation, you will be expected to show the steps that go into arriving at the solution, along with the exact solution (in terms of fundamental constants such as π, $\sqrt{3}$, etc.). If you only show bare decimal answers which come from using a calculator solving capability, you will not receive any credit. Unless specified otherwise, you should give exact answers.
- The point value for each problem is shown in parentheses to the left.

1. Beth is standing a certain distance to the west of a valley. If we impose coordinates so that the edge of the valley is at the origin, then the valley profile is given by $y = x(x-3)$ where x and y are in *hundreds of feet*. Beth throws a rock which travels along the curve $y = -\frac{1}{2}x^2 + \frac{3}{10}x + \frac{2}{5}$.



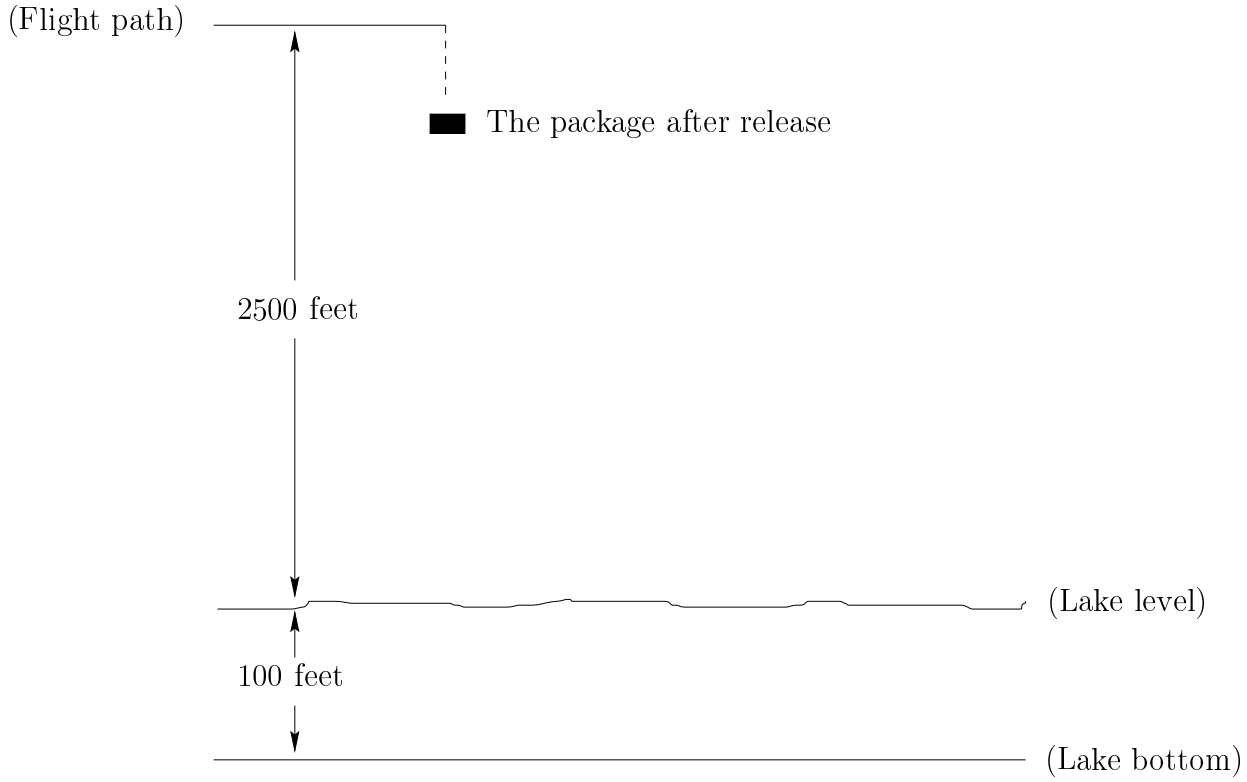
(4) (a) When the rock leaves Beth's hand it is 6 feet above the ground. What is the x-coordinate of the rock when this happens? (First express 6 feet in hundreds of feet. Give your answer in units of feet.)

(3) (b) Let $h(x)$ denote the height of the rock ABOVE THE VALLEY. Give an explicit formula for $h(x)$.

(3) (c) Find the maximum height of the rock above the valley.

(4) (d) Give the x and y coordinates of the rock when it lands in the valley. (To get the y-coordinate, it's okay to use an approximation for x.)

2. An airplane is flying at the constant altitude of 2500 above a lake which is 100 feet deep. While the plane is over the lake, a package is dropped and falls along a straight line path. the height of the package measured in feet, above the lake after t seconds is $2500 - 16t^2$. After the package enters the water, it falls at the much slower rate of 6 inches per second until it reaches the bottom. Keep in mind throughout this problem that the time variable t is measured from the instant the package is dropped from the plane.



Impose coordinates with the lake level as the x-axis and the trajectory of the package as y-axis.

(2) (a) How long does it take the package to reach the water?

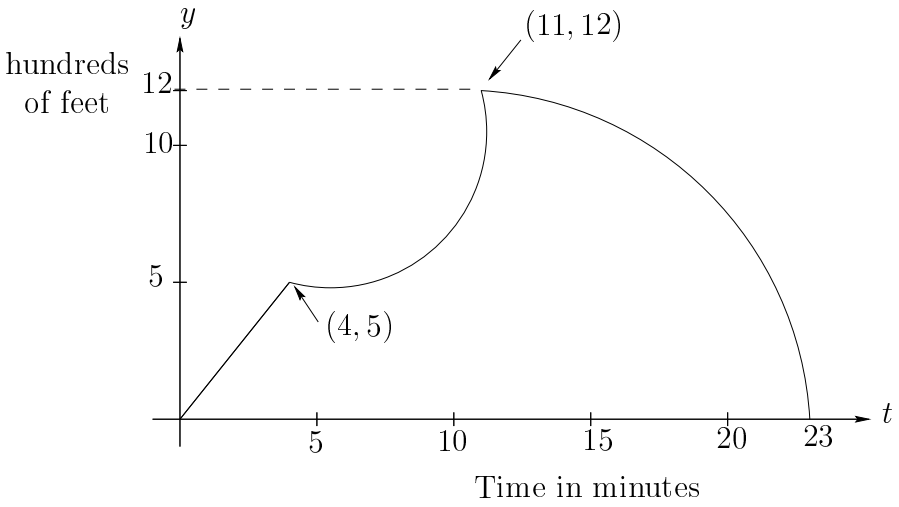
(2) (b) Let $h(t)$ be the height of the package ABOVE THE BOTTOM OF THE LAKE after t seconds. Write a formula for $h(t)$ BEFORE it enters the water.

(3) (c) Write a formula for $h(t)$ AFTER it enters the water. (Remember that downward is negative.)

(4) (d) After the package enters the water, how long does it take before it reaches the bottom of the lake?

(4) (e) Write a two part function for $h(t)$ which is valid between $t = 0$ (the time the package is released from the plane) and when the package reaches the bottom of the lake.

3. The height of a balloon above the ground is shown in the graph below. The graph shows the height $h(t)$, in hundreds of feet, after t minutes, and consists of a straight line segment and two quarter circles.



(3) (a) Find formulas in terms of time t for the height during the first 4 minutes, the time interval between 4 minutes and 11 minutes, and the last 12 minutes.

(4) (b) Write a multipart function which gives the height as a function of time during the entire flight of the balloon.

(4) (c) How long did the balloon spend above 800 feet?

4. For this problem the functions f, g, and h are defined as follows:

$$\begin{aligned} f(x) &= 2x - 3 \\ g(u) &= \sqrt{1 - u^2} \\ h(t) &= 2t^2 - t \end{aligned}$$

(3) (a) Compute the composition $g(f(x))$.

(4) (b) Give the largest possible domain so that the composition $y = g(f(x))$ is defined.

(3) (c) Find $\dfrac{h(3+z) - h(3)}{z}$. Simplify your answer.