Math 120
Worksheet 3 Solutions

(a,b)

$$h(t) = \begin{cases} \frac{7}{5}t \text{ when } 0 \le t \le 5 \\ 16 - \sqrt{81 - (t-5)^2} \text{ when } 5 \le 5 \le 14 \\ -\frac{8}{3}(t - 20) \text{ when } 14 \le t \le 20 \end{cases}$$

The formula for $h(t)$ when $t \le t \le 14$ comes from $(y - 16)^2 + (t - 5)^2 = 81$.

(c) During its ascent, the kite first reaches an altitude of 1000 feet when t satisfies

$$16 - \sqrt{81 - (t-5)^2} = 10 \text{ and } 5 \le t \le 14.$$

The solution of this equation is $t = 5 + \sqrt{45} \approx 11.71$ minutes. On its way down, the kite returns to the 1000 ft. level when

$$-\frac{8}{3}(t - 20) = 10$$

or $t = \frac{65}{4} = 16.25$ minutes. So the time spent above 1000 ft. is $\sqrt{45} - \frac{45}{4} \approx 4.54$ minutes.

(d) The change in the height between 5 and 10 minutes is

$$\begin{aligned} h(10) - h(5) &= 16 - \sqrt{56} - 7 \\ &= 9 - \sqrt{56} \text{ hundred ft.} \\ &\approx 1.52 \text{ hundred ft.} \\ &\approx 152 \text{ ft.} \end{aligned}$$

The average rate of ascent during this time is

$$\begin{aligned} \frac{h(20) - h(5)}{10 - 5} &= \frac{9 - \sqrt{56}}{5} \text{ hundred ft/min} \\ &\approx 30.4 \text{ ft/min.} \end{aligned}$$

The change in the height between 12 and 18 minutes is

$$\begin{aligned} h(18) - h(12) &= -\frac{8}{3}(18 - 20) - (16 - \sqrt{32}) \\ &= \sqrt{32} - \frac{32}{3} \text{ hundred ft} \\ &\approx -5 \text{ hundred ft.} \end{aligned}$$

The average rate of descent during this interval of time is

$$\begin{aligned} \frac{h(18) - h(12)}{18 - 12} &= \frac{1}{6}\left(\sqrt{32} - \frac{32}{3}\right) \text{ hundred ft/min} \\ &\approx -\frac{500}{6} \text{ ft/min} \\ &\approx -83.33 \text{ ft/min.} \end{aligned}$$