1. Recall that $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{1}{2x+3}$ for this problem.

(a) To find $f(g(x))$, we compute:

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{2x+3}\right) \\ &= \frac{1}{\frac{1}{2x+3}+1}. \end{aligned}$$

To simplify this, we multiply the top and bottom of the (larger) fraction by $2x+3$:

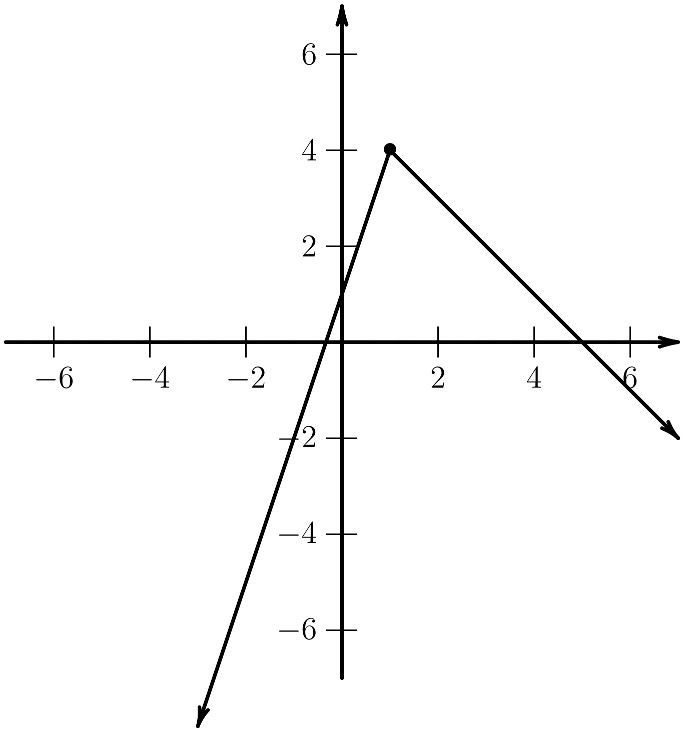
$$\begin{aligned} f(g(x)) &= \frac{1}{\frac{1}{2x+3}+1} \cdot \frac{2x+3}{2x+3} \\ &= \frac{2x+3}{1+1(2x+3)} \\ &= \frac{2x+3}{2x+4}. \end{aligned}$$

(b) The domain of $f(g(x))$ is the set of x which are in the domain of g and for which $g(x)$ is in the domain of f. The domain of f is $\{x : x \neq -1\}$, and the domain of g is $\{x : x \neq -3/2\}$. Thus x is in the domain of $f(g(x))$ if $x \neq -3/2$ and $g(x) \neq -1$. Solving $g(x) = -1$, we see this occurs when $x = -2$, so the domain of $f(g(x))$ is $\{x : x \neq -3/2, -2\}$.

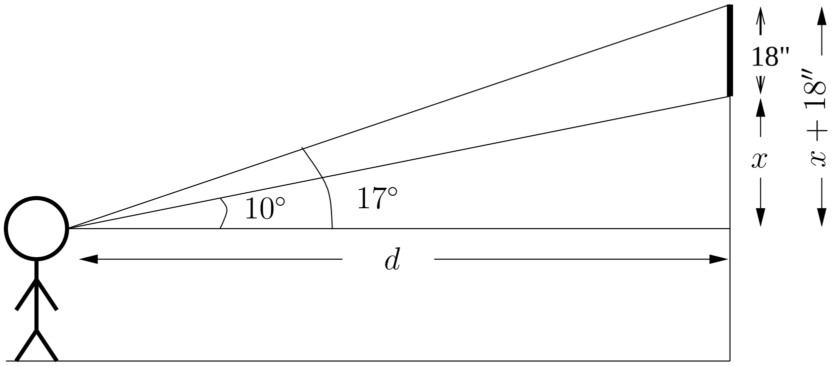
2. The graph of the multipart function

$$h(x) = \begin{cases} 3x+1 & x < 1 \\ 5-x & x \geq 1 \end{cases}$$

is two rays, or "half-lines":



3. In the picture (to the right), we have added the labels x and d and $x + 18$. We are trying to find the quantity d.



We have two right triangles, from which we get the equations

$$\tan(17^\circ) = \frac{x+18}{d} \qquad \text{and} \qquad \tan(10^\circ) = \frac{x}{d}.$$

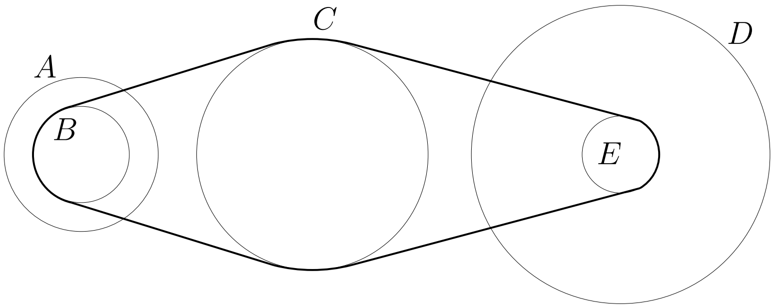
From these equations we see that $x = d\tan(10^\circ)$, so

$$\tan(17^\circ) = \frac{d\tan(10^\circ) + 18}{d}.$$

We then solve for d:

$$d = \frac{18}{\tan(17^\circ) - \tan(10^\circ)} \approx 139.10 \text{ inches.}$$

4. We reproduce the picture again for convenience.



We will repeatedly use the formula $v = r\omega$. To distinguish between the wheels we will use subscripts, so for example v_A will be the linear velocity of wheel A. The information we are given is as follows. The radii of each wheel is given:

$$r_A = 8 \text{ inches}, \qquad r_B = 5 \text{ inches}, \qquad r_C = 12 \text{ inches},$$

$$r_D = 16 \text{ inches}, \qquad \text{and} \qquad r_E = 4 \text{ inches}.$$

Since wheels A and B (and D and E) are rigidly fastened to the same axle, we have

$$\omega_A = \omega_B \qquad \text{and} \qquad \omega_D = \omega_E.$$

Wheels B, C, and E are all connected by a belt, their linear velocities are all the same:

$$v_B = v_C = v_E.$$

Finally, we are told that $\omega_A = 30$ RPM.

(a) In this part we are asked to find v_C, the linear velocity of wheel C, in inches per second. As mentioned above, this is simply a repeated application of the formula $v = r\omega$:

$$\begin{aligned}\omega_A &= 30\frac{\text{revs}}{\text{mins}} = \left(30\frac{\text{revs}}{\text{mins}}\right)\left(\frac{1\text{ min}}{60\text{ secs}}\right)\left(\frac{2\pi\text{ rads}}{1\text{ rev}}\right) = \pi\text{rads/sec}\\ \omega_B &= \omega_A = \pi\text{ rads/sec}\\ v_B &= r_B\omega_B = (5\text{ inches})\,(\pi\text{ rads/sec}) = 5\pi\text{ in/sec}\\ v_C &= v_B = 5\pi\text{ in/sec} \approx 15.71\text{ in/sec.}\end{aligned}$$

(b) Now we're asked for ω_D, the angular velocity of wheel D, in radians per second. We continue as in part (a):

$$\begin{aligned}v_E &= v_C = 5\pi\text{ in/sec}\\ \omega_E &= \frac{v_E}{r_E} = \frac{5\pi\text{ in/sec}}{4\text{ in}} = \frac{5\pi}{4}\text{ rads/sec}\\ \omega_D &= \omega_E = \frac{5\pi}{4}\text{ rads/sec} \approx 3.93\text{ rads/sec.}\end{aligned}$$

5. For $f(x) = \dfrac{1}{x-2}$, we compute

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{x+h-2}-\frac{1}{x-2}}{h} = \frac{1}{h}\left(\frac{1}{x+h-2}-\frac{1}{x-2}\right)\\ &= \frac{1}{h}\left(\frac{(x-2)}{(x+h-2)(x-2)}-\frac{(x+h-2)}{(x+h-2)(x-2)}\right)\\ &= \frac{1}{h}\cdot\frac{x-2-(x+h-2)}{(x+h-2)(x-2)}\\ &= \frac{1}{h}\cdot\frac{-h}{(x+h-2)(x-2)}\\ &= -\frac{1}{(x+h-2)(x-2)}.\end{aligned}$$

6. In this problem, $f(x) = 3e^{2x} + 4$.

(a) To find $f^{-1}(x)$, we set $y = 3e^{2x} + 4$ and solve for x. (If you prefer, you can switch x and y and solve for y.) We get $\frac{y-4}{3} = e^{2x}$, or $x = \frac{1}{2}\ln((y-4)/3)$. From this we get $f^{-1}(x) = \frac{1}{2}\ln((x-4)/3)$.

(b) The domain of $f^{-1}(x)$ is the set of all x for which $(x-4)/3 > 0$, or $\{x : x > 4\}$.

7. (a) We are looking for an exponential model $V(t) = V_0 b^t$ or $V(t) = V_0 e^{kt}$ for the dollar value $V(t)$ of the painting in the year $1947 + t$. We know that in 1947 ($t = 0$) the value was \$84,000, and in 1987 ($t = 40$), the value was \$53,900,000. That is, we're told that

$$84,000 = V_0 \cdot b^0 \qquad\text{and}\qquad 53,900,000 = V_0 \cdot b^{40}.$$

The first equation tells us $V_0 = 84.000$, and plugging this into the second equation gives $b^{40} = \frac{53,900,000}{84,000} = \frac{1925}{3}$, so $b = \sqrt[40]{\frac{1925}{3}}$ or $\left(\frac{1925}{3}\right)^{1/40}$. Hence our answer is $V(t) = 84,000\left(\frac{1925}{3}\right)^{t/40}$ (or, using the other expression, $V(t) = 84,000e^{\ln(1925/3)t/40}$).

(b) The value of the painting in 1960 ($t = 1960 - 1947 = 13$) is $V(13) = 84,000\left(\frac{1925}{3}\right)^{13/40} \approx$ \$686,522.64.

(c) The question asks for the value of t when $V(t) = \$99,000,000$. That is, we wish to solve the equation

$$84,000\left(\frac{1925}{3}\right)^{t/40} = 99,000,000$$

for t. We divide both sides by 84,000, then take the natural log of both sides to obtain

$$\frac{t}{40}\ln\left(\frac{1925}{3}\right) = \ln\left(\frac{99,000,000}{84,000}\right).$$

From this we solve for t to get

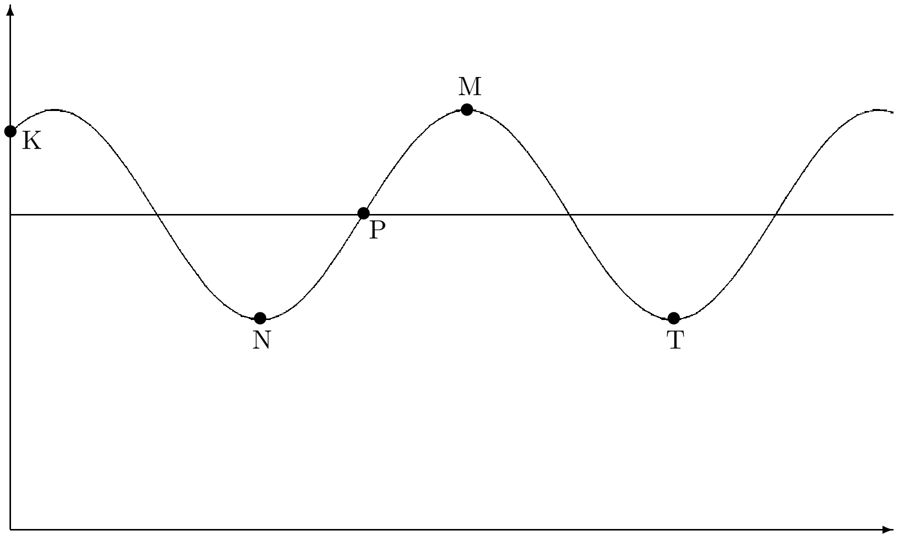
$$t = \frac{40\ln\left(\frac{99,000,000}{84,000}\right)}{\ln\left(\frac{1925}{3}\right)} \approx 43.76 \text{ years.}$$

8. (a) Recall that we are told that the population of salmon in a river has been modeled by the sinusoidal function

$$f(t) = 100\sin\left(\frac{2\pi}{7}(t-6)\right) + 300,$$

where t is in units of days. We can just read off the solution to this first part: the amplitude is $A = 100$, the period is $B = 7$, a phase shift is $C = 6$ (although we could add or subtract any whole multiple of the period, 7), and the mean is $D = 300$.

(b) We reproduce the graph here for convenience:



We've drawn in the mean line ($y = 300$) and a point P. From the phase shift, we know that the coordinates of P are $(6, 300)$. The minimum N is one quarter period before P, so it occurs at $t = 6 - B/4 = 6 - 7/4 = 4.25$, and the minimum y value is $D - A = 300 - 100 = 200$. Thus N has coordinates $(4.25, 200)$. The other minimum, T, is one period (7 days) later, at $(11.25, 200)$.

The point K has t-coordinate 0, so the y-coordinate is simply $f(0) = 100\sin\left(\frac{2\pi}{7}(0-6)\right) + 300 \approx 378.18$. Thus K has coordinates $(0, 378.18)$. Finally, the t-coordinate of the maximum M is one quarter period after P, or at $t = 6 + B/4 = 6 + 7/4 = 7.75$. The y-coordinate is $D + A = 300 + 100 = 400$, so M has coordinates $(7.75, 400)$.

(c) To find the total amount of time (during the first 15 days) the salmon population is at least 250, we first find when the population is precisely 250. That is, we solve the equation

$$100\sin\left(\frac{2\pi}{7}(t-6)\right) + 300 = 250,$$

or

$$\sin\left(\frac{2\pi}{7}(t-6)\right) = -\frac{1}{2}.$$

We take the inverse sine to see that

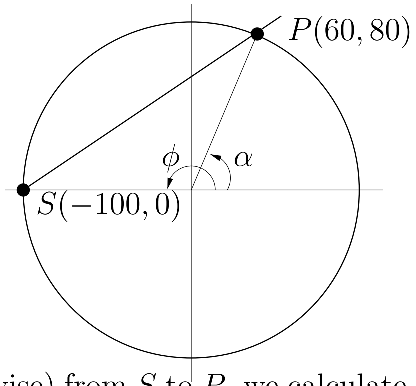
$$\frac{2\pi}{7}(t-6) = \sin^{-1}(-1/2) = -\frac{\pi}{6}$$

(this is the principal solution), or

$$\frac{2\pi}{7}(t-6) = \pi - \sin^{-1}(-1/2) = \pi - -\frac{\pi}{6} = \frac{7\pi}{6}$$

(this is the symmetry solution). Solving for t in these equations, we get $t = 6 - \frac{7}{12} = \frac{65}{12} \approx 5.4167$ and $t = 6 + \frac{49}{12} = \frac{121}{12} \approx 10.0833$. Adding and subtracting periods (7 days), we see that there were at least 250 salmon from $t = 0$ (actually $t = -\frac{19}{12} \approx -1.5833$, but we're looking for numbers between 0 and 15) to $t = \frac{37}{12} \approx 3.0833$ (a period of $\frac{37}{12} \approx 3.0833$ days), from $t = \frac{65}{12}$ to $t = \frac{121}{12} \approx 10.0833$ (a period of $\frac{56}{12} = \frac{14}{3} \approx 4.6667$ days), and from $t = \frac{149}{12} \approx 12.4167$ to $t = 15$ (again, actually $t = \frac{205}{12} \approx 17.0833$, but we stop at 15) (a period of $\frac{31}{12} \approx 2.5833$ days). The total time is thus $\frac{37}{12} + \frac{56}{12} + \frac{31}{12} = \frac{31}{3} \approx 10.3333$ days.

9. We have redrawn the track, with a few angles added and labeled. Recall that that the radius of the track is 100 feet. Both Jackie and Lee start at $S = (-100, 0)$, but Jackie runs around the track, while Lee jogs along the line drawn, from S to $P = (60, 80)$.



(a) To find how long it takes Jackie to run (clockwise) from S to P, we calculate the distance she runs. To do this, we first find the angle she runs through by finding the angles α and ϕ, as labeled in the picture. The angle ϕ is clearly π, but α is more complicated. One way to find α is to use the fact that $\tan(\alpha)$ is the slope of the line from the origin to $P = (60, 80)$; that is, $\tan(\alpha) = \frac{80}{60} = 4/3$. Thus Jackie runs through an angle of $\theta = \pi - \tan^{-1}(4/3) \approx 2.21430$ radians.

The arc length s can now be found using the formula $s = r\theta$: Jackie has run a distance of $s = (100 \text{ feet})(2.21430) = 221.430$ feet. She runs at 10 feet per second, so this takes her 22.143 seconds.

(b) Lee covers the distance from $S = (-100, 0)$ to $P = (60, 80)$ in the same 22.143 seconds. This distance is $\sqrt{(-100-60)^2 + (0-80)^2} = 80\sqrt{5} \approx 178.885$ feet, so Lee's speed is $\frac{80\sqrt{5}\text{ feet}}{22.143\text{ sec}} \approx 8.079$ feet per second.

(c) Jackie's parametric equations are of the form

$$\begin{aligned} x(t) &= r\cos(\omega t + \theta_0) + x_c \\ y(t) &= r\sin(\omega t + \theta_0) + y_c, \end{aligned}$$

where r and (x_c, y_c) are the radius and center of the circle, ω is her angular speed, and θ_0 is her angle at time $t = 0$. We know $r = 100$ feet and the circle's center is $(x_c, y_c) = (0, 0)$. Jackie's angular speed can be found via the equation $v = r\omega$. We know her linear speed is 10 feet per second, and the radius is 100 feet; thus $\omega = -0.1$ radians per second. (This is negative because Jackie is running clockwise.) Her initial angle is $\theta_0 = \pi$, so Jackie's position at time t is given by the parametric equations

$$\begin{aligned} x(t) &= 100\cos(\pi - 0.1t) \\ y(t) &= 100\sin(\pi - 0.1t)\,. \end{aligned}$$

(d) The parametric equations for Lee's position after t seconds are of the form

$$\begin{aligned} x(t) &= x_0 + v_x t \\ y(t) &= y_0 + v_y t, \end{aligned}$$

where v_x and v_y are the horizontal and vertical speeds and (x_0, y_0) is Lee's position at $t = 0$. Lee starts at S, so $(x_0, y_0) = (-100, 0)$. The speeds can be computed by using the fact that at $t = 22.143$ seconds Lee is at $P = (60, 80)$:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{60 - -100\text{ feet}}{22.143 - 0\text{ sec}} \approx 7.23\text{ feet per second}$$

and

$$v_y = \frac{\Delta y}{\Delta t} = \frac{80 - 0\text{ feet}}{22.143 - 0\text{ sec}} \approx 3.61\text{ feet per second.}$$

Hence we get parametric equations

$$\begin{aligned} x(t) &= -100 + 7.23t \\ y(t) &= 3.61t. \end{aligned}$$