1. Let $f(x) = 2x^2 + 3x + 5$. Assuming $h \neq 0$, simplify the expression

$$\frac{f(x+h) - f(x)}{h}$$

as much as possible.

Solution:

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 + 3(x+h) + 5 - (2x^2 + 3x + 5)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h + 5 - 2x^2 - 3x - 5}{h} =$$

$$\frac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3.$$

2. An airplane is flying due north from airport A. Airport B is located 15 miles west and 300 miles north of airport A. The airplane will be on airport B's radar when the plane is within 113 miles of airport B. For what distance of its flight will the airplane be on the radar?

Solution:

We impose a coordinate system with Airport A at the origin, and Airport B at the point $(-15, 300)$. The plane's path is along $x = 0$. The circle representing Airport B's radar range is

$$(x + 15)^2 + (y - 300)^2 = 113^2.$$

We find the intersection of this circle with $x = 0$:

$$(0 + 15)^2 + (y - 300)^2 = 113^2$$

$$(y - 300)^2 = 113^2 - 15^2 = 12544 = 112^2$$

$$y - 300 = \pm 112$$

$$y = 300 \pm 112.$$

So the intersection points are $(0, 188)$ and $(0, 412)$, and the distance that the plane will be on the radar is 412-188=224 miles.

3. On a certain island in the year 1850 there were 1040 sparrows. A survey in 1900 showed there to be 2100 sparrows. Assuming that the population of sparrows always changes as a linear function of time, answer the following two questions:

(a) How many sparrows were there in the year 1935?

(b) In what year will there be 3000 sparrows on the island?

Solution: The two data points given can be placed into a coordinate system as the points $(1850, 1040)$ and $(1900, 2100)$. Finding the slope of the line through these points, we get
$$m = \frac{2100 - 1040}{1900 - 1850} = \frac{1060}{50} = 21.2$$
so the equation of the line through the points is
$$y - 1040 = 21.2(x - 1850)$$
i.e.,
$$y = 21.2(x - 1850) + 1040.$$

Thus, assuming a linear model, in 1935 there were $21.2(1935 - 1850) + 1040 = 2842$ sparrows.

Setting $y = 3000$ and solving for x we find
$$3000 = 21.2(x - 1850) + 1040$$
$$x = \frac{3000 - 1040}{21.2} + 1850 = 1942.452830188679...$$
So there were 3000 sparrows on the island in 1942 (or 1943).

4. A trucker is paid by the mile. For the first 60 miles of a trip, the trucker gets paid 40 cents per mile. For each mile over 60 miles and under 300 miles, the trucker gets 45 cents per mile. For miles over 300, the trucker gets 50 cents per mile. Suppose the trucker travels at a constant speed to 60 miles per hour. Write a multipart function for the amount of money the trucker makes on a trip as a function of time.

Solution:

The trucker drives 60 miles per hour, so for the first hour of a trip the trucker is paid 40 cents per mile, or $(0.4\frac{\text{dollars}}{\text{mile}})(60\frac{\text{miles}}{\text{hour}}) = 24\frac{\text{dollars}}{\text{hour}}$. From 60 to 300 miles (i.e., 1 to 5 hours) the trucker makes $(0.45\frac{\text{dollars}}{\text{mile}})(60\frac{\text{miles}}{\text{hour}}) = 27\frac{\text{dollars}}{\text{hour}}$, and over 300 miles (i.e., 5 hours) the trucker makes $(0.5\frac{\text{dollars}}{\text{mile}})(60\frac{\text{miles}}{\text{hour}}) = 30\frac{\text{dollars}}{\text{hour}}$.

Putting this together, the multipart function that gives the amount $M(t)$ of money made as a function of time is
$$M(t) = \begin{cases} 24t & \text{if } t \leq 1 \\ 24 + 27(t - 1) & \text{if } 1 < t < 5 \\ 132 + 30(t - 5) & \text{if } t \geq 5. \end{cases}$$

5. Let $f(x) = 2x + 3$, and $g(x) = bx + 5$. What should b be so that $f(g(x)) = g(f(x))$ for all x?

Solution:

We have
$$f(g(x)) = f(bx + 5) = 2(bx + 5) + 3 = 2bx + 13$$
and
$$g(f(x)) = g(2x + 3) = b(2x + 3) + 5 = 2bx + 3b + 5$$
If these functions are equal for all x, then
$$3b + 5 = 13$$
so $b = 8/3$.

6. Suppose you have 300 meters to enclose a rectangular pasture. One side of the pasture will be along a straight river so requires no fencing. Also, you want to use some of the fencing to split the pasture into two parts, as illustrated. What is the largest area the pasture can have?

Solution:

Letting x be the length of the side of the pasture that is parallel to the river, and y be the length of the perpendicular side, we have
$$x + 3y = 300$$
so
$$x = 300 - 3y.$$
The area A of the pasture is xy, which we can write as
$$A = xy = y(300 - 3y) = 300y - 3y^2.$$
Completing the square yields
$$A = -3y^2 + 300y = -3(y^2 - 100y) = -3((y - 50)^2 - 2500) = -3(y - 50)^2 + 7500$$
from which we see that the maximum area the pasture can have is 7500 square meters.