$\qquad$

## How Much Soap Can You Buy with Two Million Dollars?

You're in your new job as sales analyst for VariKleen Soap, Inc, when your bosses tell you, "We need a profit analysis on our new line of Very VariKleen soap." You have the following data: If you charge $\$ 1$ per bar of soap, you'll sell 2 million bars. If you charge $\$ 2$ per bar, you'll sell 1.5 million bars of soap.

1. Make a linear model $p(q)$ of the price, $p$, you should sell the soap at if you want to sell q million bars. (Note that q will be in millions of bars. You'll have values like $\mathrm{q}=2$ or $\mathrm{q}=1.5$ and NOT values like $\mathrm{q}=2,000,000$ or $\mathrm{q}=1,500,00$.
2. Your revenue, $R$, is the total amount of money you take in, in this case from selling soap. It's given by the formula $\mathrm{R}=\mathrm{qp}$, where q is the number of items sold and p is the price you sell them at. Think about why this makes sense, and then plug your answer from part 1 into this formula to get rid of the $p$ in the above formula. You should get $R(q)$, a function of only $q$. This function tells you how much money you'll bring in if you sell q million items.
3. Find the values of $q$ where your revenue is zero. Do these make sense? Graph $R(q)$ on the graph on the next page with q along the x -axis and millions of dollars along the y axis.
4. Your profit, P , is your revenue minus expenses. It's given by the formula $\mathrm{P}=\mathrm{R}-\mathrm{C}$, where C are the costs you incur.
Example: You make $\$ 400$ a week at your job, but have to pay $\$ 350$ in bills every week. Your revenue is $\mathrm{R}=400$. Your costs are $\mathrm{C}=350$. Your profit (how much you've made) is $P=R-C=400-350=50$ dollars. In the case of VariKleen Soap, the costs you incur are due to production, shipping, advertising, etc. You know that your costs are given by the function $C(q)=F I X M E E E E E E E E E E E q^{2}-13 q+15 . \mathrm{C}(\mathrm{q})$ GOES NEGATIVE!!!!! YUCK!!! Find the formula $\mathrm{P}(\mathrm{q})$ for your profit. (Be careful not to get the price, p , mixed up with the Profit, P.)

5. Using the fact that $\mathrm{C}(\mathrm{q})$ is a parabola, graph $\mathrm{C}(\mathrm{q})$ on the same graph as R .
6. Using the vertex formula, find the $q$ where you attain your maximum profit. Draw the vertical line on the graph that corresponds to this value of $q$. If you drew $R(q)$ and $C(q)$ right, this should be a special point in some way. How is it special? Can you say why the maximum profit is at the q value where this happens?
