

$$\textcircled{1} \text{ (a) } y = \frac{ax+b}{x+d}$$

$$\textcircled{1} x=2, y=35 \Rightarrow 35 = \frac{2a+b}{2+d}$$

$$\textcircled{2} x=10, y=55 \Rightarrow 55 = \frac{10a+b}{10+d}$$

$$\textcircled{3} \text{ horizontal asymptote at } y=64 \Rightarrow \textcircled{a=64}$$

COMBINE/SIMPLIFY

$$\textcircled{1} 70 + 35d = 128 + b$$

$$\textcircled{2} 550 + 55d = 640 + b$$

$$\textcircled{3} - \textcircled{1} \Rightarrow 480 + 20d = 512$$

$$20d = 32$$

$$d = \frac{32}{20} = \frac{8}{5} = 1.6$$

BACK
SUBSTITUTE

$$70 + 35(1.6) = 128 + b$$

$$\Rightarrow 126 = 128 + b \Rightarrow \textcircled{b=-2}$$

$$y = \frac{64x-2}{x+1.6}$$

$$\text{check: } x=2 \Rightarrow y = \frac{64(2)-2}{(2)+1.6} = 35 \checkmark$$

$$x=10 \Rightarrow y = \frac{64(10)-2}{(10)+1.6} = 55 \checkmark$$

$$\text{(b) } 50 = \frac{64x-2}{x+1.6}$$

$$50x + 80 = 64x - 2$$

$$82 = 14x$$

$$x = 82/14 = 4\frac{1}{7}$$

$$x = 5.857142$$

$$x \approx 5.86 \text{ years old}$$

$$(2) (a) \quad y = m(x - x_1) + y_1 \quad m = \frac{300 - 285}{205 - 215} = -\frac{15}{10} = -\frac{3}{4}$$

$$y = -\frac{3}{4}(x - 205) + 300 = -\frac{3}{4}x + \frac{615}{4} + 300$$

$$y = -\frac{3}{4}x + \frac{1815}{4} = -0.75x + 453.75$$

$$\text{Revenue} = R(x) = (\text{price per room}) (\text{rooms}) = xy$$

$$R(x) = x(-0.75x + 453.75)$$

$$R(x) = -0.75x^2 + 453.75x$$

DOWNWARD FACING PARABOLA ↗

MAXIMUM OCCURS AT THE VERTEX.

$$-0.75x = -\frac{b}{2a} = -\frac{453.75}{2(-0.75)} = 302.50 \text{ per room}$$

$$\text{MAX REVENUE} = R(302.50)$$

$$= -0.75(302.5)^2 + 453.75(302.5)$$

$$= 68629.6875$$

$$\boxed{\$68629.69}$$

$$(b) \quad 150 = -0.75x + 453.75$$

$$-303.75 = -0.75x$$

$$x = \frac{-303.75}{-0.75} = 405$$

← BOTH GIVE CORRECT ANSWER

$$\text{Revenue} = 150 \cdot 405 = -0.75(405)^2 + 453.75(405)$$

$$= \boxed{\$60,750}$$

$$\text{PRICE} = 231.25 \text{ per room}$$

$$\text{PROFIT} = 231.25 \cdot 405 = 93637.50$$

(3) (a) AREA OF A WEDGE = $\frac{\theta}{360} \pi r^2$
 (θ IN DEGREES)

LARGE WEDGE ($r=22$) = $\frac{105}{360} \pi (22)^2 = \frac{105}{360} \pi 484 = \frac{847\pi}{6}$
 $\approx 443.488162932 \text{ in}^2$

SMALL WEDGE ($r=6$) = $\frac{105}{360} \pi (6)^2 = \frac{105\pi}{10} = \frac{21\pi}{2}$
 $\approx 32.9867228627 \text{ in}^2$

AREA SWEEP OUT = $\frac{847\pi}{6} - \frac{21\pi}{2} = \frac{413\pi}{3}$
 $\approx 443.488162932 - 32.9867228627$
 $= 410.501440069 \text{ sq. in}$
 410.50 in^2

(b) $f(x) = 3x + c$; so $f(f(x)) = f(3x + c)$
 $= 3(3x + c) + c$
 $= 9x + 3c + c$
 $= 9x + 4c$

So we want $9x + 4c = 9x - 10$
 Thus, $4c = -10 \Rightarrow c = -\frac{10}{4} = -\frac{5}{2} = -2.5$

(c) $y = \frac{(\sqrt{x}-1)^2}{3}$ $0 \leq x \leq 1$

$3y = (\sqrt{x}-1)^2$
 $\pm \sqrt{3y} = \sqrt{x} - 1$
 $1 \pm \sqrt{3y} = \sqrt{x}$

$\sqrt{x} = 1 - \sqrt{3y}$

$x = (1 - \sqrt{3y})^2 = 1 - 2\sqrt{3y} + 3y$

$f^{-1}(y) = (1 - \sqrt{3y})^2$ $= 1 - 2\sqrt{3y} + 3y$

④ (a) $B(x) = y_0 b^x$

① $x=0, y=1250 \Rightarrow y_0 = 1250$

② $x=5, y=2y_0 \Rightarrow 2500 = 1250 b^5$
 $2 = b^5$

$b = 2^{1/5} \approx 1.148698355$

$B(x) = 1250 (2)^{x/5} = 1250 (1.148698355)^x$

$15000 = 1250 (2)^{x/5}$

$12 = 2^{x/5}$

$\ln(12) = \frac{x}{5} \ln(2) \Rightarrow \frac{\ln(12)}{\ln(2)} = \frac{x}{5} \Rightarrow x = 5 \frac{\ln(12)}{\ln(2)}$

$x \approx 17.9248125036$

$x \approx 17.92$ years after 2000

(b) $D(x) = y_0 b^x$

① $x=3, y=9000 \Rightarrow 9000 = y_0 b^3$

② $x=6, y=8000 \Rightarrow 8000 = y_0 b^6$

COMBINE/SIMPLIFY

②/① $\Rightarrow \frac{8000}{9000} = \frac{y_0 b^6}{y_0 b^3} \Rightarrow \frac{8}{9} = b^3 \Rightarrow b = \left(\frac{8}{9}\right)^{1/3} \approx 0.9614997$

BACK SUBSTITUTE

$y_0 = \frac{9000}{b^3} = \frac{9000}{(8/9)} = 10125$

$D(x) = 10125 \left(\frac{8}{9}\right)^{x/3} = 10125 (0.961499713538)^x$

$B(x) = 2D(x)$

$1250 (2)^{x/5} = 2 \cdot 10125 \left(\frac{8}{9}\right)^{x/3}$

$\frac{2^{x/5}}{\left(\frac{8}{9}\right)^{x/3}} = \frac{20250}{1250} \Rightarrow \left(\frac{2^{1/5}}{\left(\frac{8}{9}\right)^{1/3}}\right)^x = \frac{20250}{1250}$

$\Rightarrow (1.1946944329)^x = 16.2$

$x \ln(1.1946944329) = \ln(16.2)$

$x = \frac{\ln(16.2)}{\ln(1.1946944329)} \approx 15.6557660929$ years after 2000

15.66 years after 2000

2015