Math 120 - Autumn 2016
Final Exam
December 10, 2016

Name: ______________________ Student ID no. : ______________

Signature: ______________________________ Section: __________

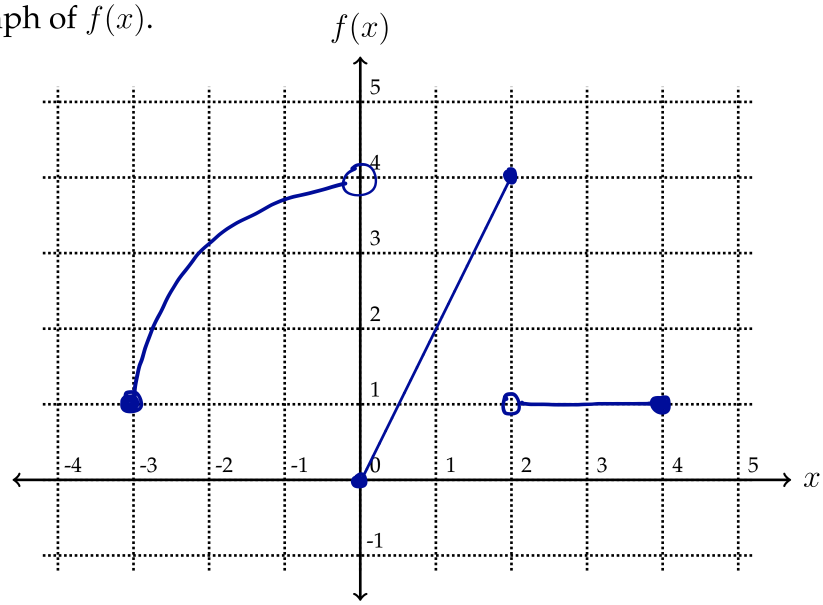
1	15	
2	16	
3	14	
4	14	
5	15	
6	14	
7	12	
Total	100	

- This exam consists of SEVEN problems on EIGHT pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 170 minutes to complete the exam.

1. **[5 points per part]** For parts (a) through (c), consider the following multipart function:

$$f(x) = \begin{cases} 1+\sqrt{9-x^2} & \text{if } -3 \le x < 0 \\ 2x & \text{if } 0 \le x \le 2 \\ 1 & \text{if } 2 < x \le 4 \end{cases}$$

(a) Sketch a graph of $f(x)$.



(b) Find all values of x such that $f(x) = -2x + 2.2$.

$-3 \le x < 0$:

$1+\sqrt{9-x^2} = -2x+2.2$

$\sqrt{9-x^2} = -2x+1.2$

$9-x^2 = 4x^2-4.8x+1.44$

$0 = 5x^2-4.8x-7.56$

$x = \dfrac{4.8 \pm \sqrt{4.8^2+151.2}}{10}$

$= \cancel{1.8}$ or $\boxed{-0.84}$

$0 \le x \le 2$:

$2x = -2x+2.2$

$4x = 2.2$

$x = \boxed{0.55}$

$2 < x \le 4$:

$1 = -2x+2.2$

$1.2 = 2x$

$x = \cancel{0.6}$

Two solutions

(c) Let $g(x)$ be the function found by taking the graph of $f(x)$ and shifting it 1 unit left. Write the multipart rule for $g(x)$.

- Replace x w/ $x+1$.
- Shift intervals left.

$$\boxed{g(x) = \begin{cases} 1+\sqrt{9-(x+1)^2} & \text{if } -4 \le x < -1 \\ 2(x+1) & -1 \le x \le 1 \\ 1 & 1 < x \le 3 \end{cases}}$$

2. (a) **[4 points]** A mysterious red dot is moving through the xy-plane at a constant speed.

At time $t = 0$, it starts at $(-1, -1)$. It moves in a straight line towards the point $(31, 13)$, reaching it in 10 seconds.

Write parametric equations for the red dot's coordinates after t seconds.

$x_0 = -1$ $\quad y_0 = -1$

$x_1 = 31$ $\quad y_1 = 13$

$\Delta x = 32$ $\quad \Delta y = 14$

$\Delta t = 10$

$$\boxed{\begin{aligned} x &= -1 + 3.2t \\ y &= -1 + 1.4t \end{aligned}}$$

(b) **[5 points]** Fungo is also moving in the xy-plane. At time $t = 0$, he starts at $(2, 6)$.

Fungo runs in a straight line towards $(8, -2)$ at a speed of 2 units per second.

Write parametric equations for Fungo's coordinates after t seconds.

$x_0 = 2$ $\quad y_0 = 6$

$x_1 = 8$ $\quad y_1 = -2$

$\Delta x = 6$ $\quad \Delta y = -8$

$$\Delta t = \frac{\text{dist}}{\text{speed}} = \frac{\sqrt{6^2 + 8^2}}{2} = 5$$

$$\boxed{\begin{aligned} x &= 2 + 1.2t \\ y &= 6 - 1.6t \end{aligned}}$$

(c) **[7 points]** When is Fungo closest to the red dot?

$$\text{dist} = \sqrt{\big((-1+3.2t) - (2+1.2t)\big)^2 + \big((-1+1.4t) - (6-1.6t)\big)^2}$$

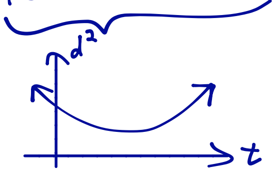
$$= \sqrt{(-3+2t)^2 + (-7+3t)^2}$$

$$= \sqrt{9 - 12t + 4t^2 + 49 - 42t + 9t^2}$$

$$= \sqrt{\underbrace{13t^2 - 54t + 58}_{d^2}} \quad \longleftarrow \text{Quadratic,}$$

minimum when $t = h = \dfrac{-b}{2a} = \dfrac{54}{26}$

$$\approx \boxed{2.077 \text{ sec.}}$$



3. The population of Threeattle triples every ten years.

Four years from now, there will be 10,000 more people in Threeattle than there are today.

(a) **[6 points]** Write a function $f(x)$ for the population of Threeattle x years from today.

$f(x) = A_0 b^x$

$b^{10} = 3 \rightarrow b = 3^{1/10}$

$f(4) = f(0) + 10000$

$A_0 b^4 = A_0 + 10000$

$A_0(b^4 - 1) = 10000$

$A_0 = \dfrac{10000}{b^4 - 1} = 18121$

$$\boxed{f(x) = 18121\left(3^{1/10}\right)^x}$$

(b) **[6 points]** Compute the inverse of the function you found in part (a).

$y = 18121\left(3^{1/10}\right)^x$

$\dfrac{y}{18121} = \left(3^{1/10}\right)^x$

$\ln\left(\dfrac{y}{18121}\right) = \ln\left(\left(3^{1/10}\right)^x\right)$

$\ln\left(\dfrac{y}{18121}\right) = x \ln\left(3^{1/10}\right)$

$x = \dfrac{\ln\left(\frac{y}{18121}\right)}{\ln\left(3^{1/10}\right)}$

$$\boxed{f^{-1}(y) = \dfrac{\ln\left(\frac{y}{18121}\right)}{\ln\left(3^{1/10}\right)}}$$

(c) **[2 points]** In one sentence, explain the meaning of the inverse function you found.

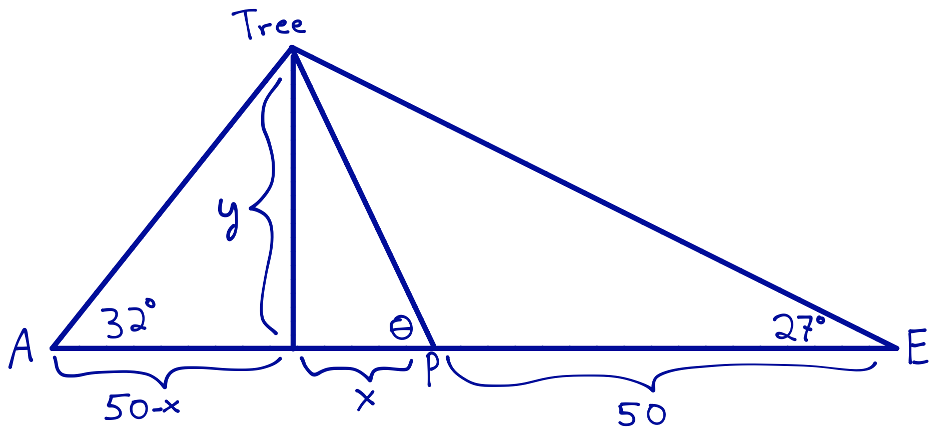
$f^{-1}(y)$ is the year when the population will reach y.

4. **[14 points]** Angelica and Eliza are standing 100 meters apart, and Peggy is standing exactly halfway between them. Somewhere else in between them is a tree.

From where they stand, Angelica, Eliza, and Peggy all measure the angle of elevation of the top of the tree above the ground.

Angelica measures it to be $32°$. Eliza measures it to be $27°$.

What measurement does Peggy get?



$$\tan(32°) = \frac{y}{50-x} \longrightarrow y = (50-x)\tan(32°)$$

$$\tan(27°) = \frac{y}{50+x} \longrightarrow y = (50+x)\tan(27°)$$

$$(50-x)\tan(32°) = (50+x)\tan(27°)$$

$$50\tan(32°) - 50\tan(27°) = x\tan(32°) + x\tan(27°)$$

$$x = \frac{50(\tan(32°) - \tan(27°))}{\tan(32°) + \tan(27°)} \approx 5.0839$$

$$y = (50+x)\tan(27°) \approx 28.0667$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \approx \boxed{79.7329°}$$

5. $f(x)$ is a linear-to-linear rational function whose graph has a horizontal asymptote of $y = 5$ and passes through the points $(1, -10)$ and $(2, -35)$.

(a) **[7 points]** Write a formula for $f(x)$.

$$f(x) = \frac{ax+b}{x+d}$$

$$-10 = \frac{a+b}{1+d} \longrightarrow -10 - 10d = a + b \longrightarrow b = -10 - 10d - a$$

$$-35 = \frac{2a+b}{2+d} \longrightarrow -70 - 35d = 2a + b \longrightarrow -70 - 35d = 10 - 10 - 10d - 5$$

$$-65 = 25d$$

$$\boxed{d = -2.6}$$

$$\boxed{a = 5}$$

$$b = -10 + 26 - 5$$

$$\boxed{b = 11}$$

$$\boxed{f(x) = \frac{5x+11}{x-2.6}}$$

(b) **[2 points]** What is the domain of $f(x)$?

Everything but 2.6:

$$\boxed{(-\infty, 2.6) \cup (2.6, \infty)}$$

(c) **[6 points]** Let $g(x) = f(f(x))$. Find the asymptotes of $g(x)$.

$$g(x) = f(f(x)) = \frac{5\left(\frac{5x+11}{x-2.6}\right) + 11}{\left(\frac{5x+11}{x-2.6}\right) - 2.6} \cdot \frac{(x-2.6)}{(x-2.6)} = \frac{25x + 55 + 11x - 28.6}{5x + 11 - 2.6x + 6.76}$$

$$g(x) = \frac{36x + 26.4}{2.4x + 17.76} \quad \frac{\div 2.4}{\div 2.4} = \frac{15x + 11}{x + 7.4}$$

asymptotes: $\boxed{\begin{array}{l} y = 15 \\ x = -7.4 \end{array}}$

6. **[14 points]** The temperature in Meereen is a sinusoidal function of time.

The temperature will decrease for the next 5 years until it reaches a minimum of $20°$. Then the temperature will climb until 21 years from now, when it reaches a maximum of $80°$.

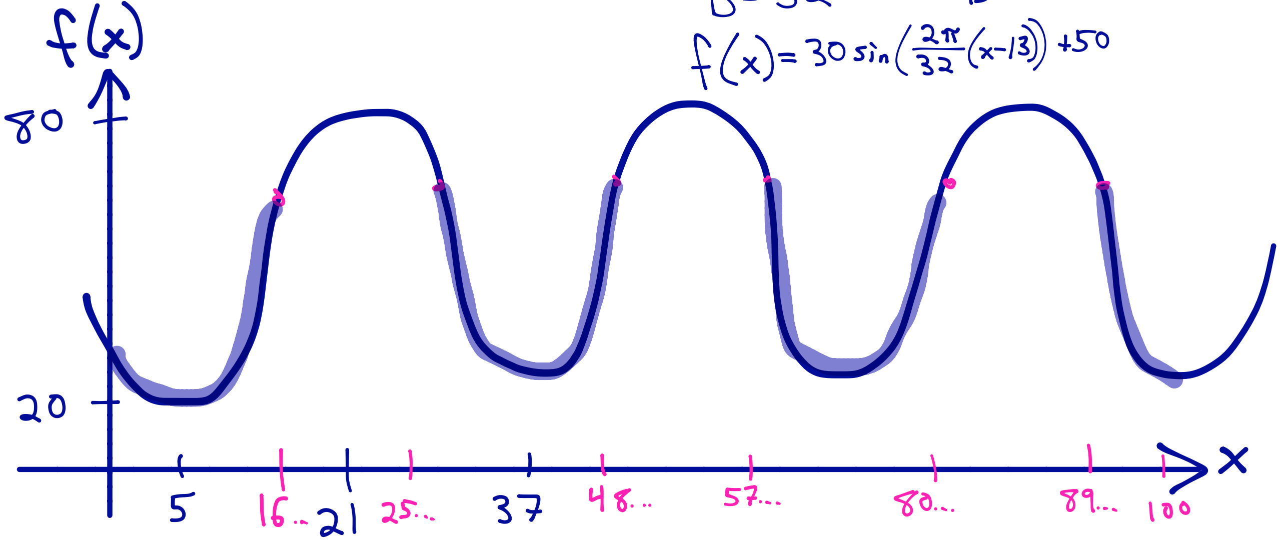
Dany is unhappy when the temperature in Meereen is below $70°$. Over the next 100 years, for how long will she be unhappy?

$f(x)$ = temp after x years.

$A = 30$ $\quad C = 13$

$B = 32$ $\quad D = 50$

$f(x) = 30\sin\left(\frac{2\pi}{32}(x-13)\right) + 50$



$30\sin\left(\frac{2\pi}{32}(x-13)\right) + 50 = 70$

$\sin\left(\frac{2\pi}{32}(x-13)\right) = \frac{2}{3}$

$x = 13 + \frac{32}{2\pi}\sin^{-1}\left(\frac{2}{3}\right) = 16.716$ princ.

symmetry: $2C + \frac{B}{2} - p = 25.283$

list of sols:

16.716

25.283

48.716

57.283

80.716

89.283

+32 +32 +32

Total time: $(16.716 - 0)$

$+ (48.716 - 25.283)$

$+ (80.716 - 57.283)$

$+ (100 - 89.283)$

$= \boxed{74.299 \text{ years}}$

7. (a) **[12 points]** Essun is running 3 meters per second clockwise around a circular track.

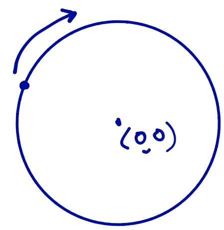
From her starting point, it takes her 9 seconds to reach the northernmost point of the track, and then an additional 13 seconds to reach the easternmost point of the track.

After 2 minutes, how far east is Essun from the westernmost point of the track?

$v = 3$ $\quad x_0 = 0$

$\omega = \dfrac{\frac{\pi}{2}}{13} = \dfrac{\pi}{26}$ $\quad y_0 = 0$

$r = \dfrac{v}{\omega} = \dfrac{78}{\pi}$ $\quad t = 120$



$\theta_0 - 9\omega = \frac{\pi}{2} \rightarrow \theta = \frac{\pi}{2} + \frac{9\pi}{26} = \frac{22\pi}{26} = \frac{11\pi}{13}$

$x = 0 + \frac{78}{\pi}\cos\left(\frac{11\pi}{13} - \frac{\pi}{26}(120)\right) \approx 18.584$

Westernmost point is at $\left(\frac{-78}{\pi}, 0\right)$, so this is

$18.584 + \frac{78}{\pi} \approx \boxed{43.412 \text{ m east}}$

(b) **[0 points]** You're done! Please check your work, then enjoy this celebratory maze. Help Essun get to the exit. Unfortunately, after spending so much time running clockwise, she has forgotten how to turn left.