Math 120 (Pezzoli)
Fall 2019
Midterm $\#2$

Name ____________

TA: ____________

Section: ____________

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Instructions:

- Your exam contains 3 problems.
- Your exam should contain 4 pages; please make sure you have a complete exam.
- Box in your final answer.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).
- The only calculator allowed is the Ti-30x IIS.
- Round off your final answers to 2 decimal places, unless you are asked for exact answers.

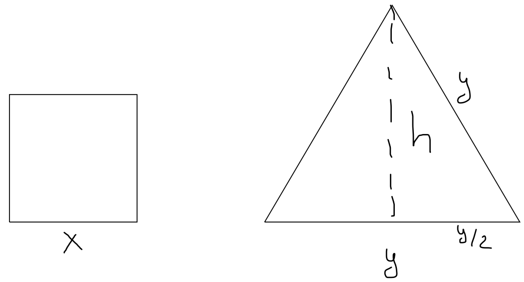
Problem $\#1$ (10 pts) ________

Problem $\#2$ (12 pts) ________

Problem $\#3$ (13 pts) ________

TOTAL (35 pts) ____________

1. You want to build two enclosures using exactly 3000 feet of fencing. One enclosure will be an equilateral triangle, the other a square. What should the side of the square be in order to minimize the area of the two combined enclosures ?



$h^2 + \left(\frac{y}{2}\right)^2 = y^2$

$h = \sqrt{y^2 - \frac{y^2}{4}} = y \cdot \frac{\sqrt{3}}{2}$

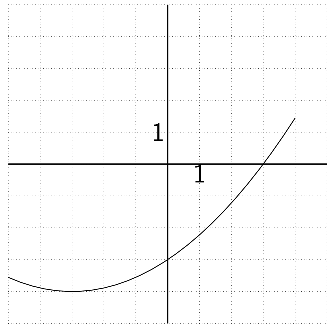
$A = x^2 + \frac{1}{2} y \cdot y \frac{\sqrt{3}}{2}$

$4x + 3y = 3000 \qquad y = 1000 - \frac{4}{3}x$

$A = x^2 + \frac{\sqrt{3}}{4}\left(1000 - \frac{4}{3}x\right)^2 = x^2 + \frac{\sqrt{3}}{4}\left(10^6 - \frac{8000}{3}x + \frac{16}{9}x^2\right) =$

$= \left(1 + \frac{4\sqrt{3}}{9}\right)x^2 - \frac{8000\sqrt{3}}{12}x + \frac{\sqrt{3}}{4}10^6$ The graph of $A(x)$ looks up $\cup$ min is at vertex $h = \dfrac{\frac{8000\sqrt{3}}{12}}{2\left(1 + \frac{4\sqrt{3}}{9}\right)} \approx 326.22$ m.

2. The function f graphed below has domain $-5 \leq x \leq 4$



a) What is the value of $f(f(0))$?

$f(-3) = -4$

b) What is the domain of $f(\frac{x}{2})$?

$-10 \leq x \leq 8$

The next two questions are unrelated to parts a), b) above. Consider the function $g(x) = 2(x-1)^2 + 4$.

c) Write a formula for the function whose graph is the graph of g shifted horizontally to the right of two units, then reflected across the y axix, then shifted vertically up of three units .

1) $2(x-3)^2+4$
2) $2(-x-3)^2+4$
3) $2(-x-3)^2+7$

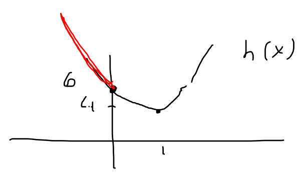
d) Let $h(x)$ be the function you obtain by restricting $g(x)$ to the domain $x \leq 0$ Find a formula for $h^{-1}(y)$, the inverse of $h(x)$, and find the domain of $h^{-1}(y)$.
DOMAIN = $[6, +\infty)$

$y = 2(x-1)^2+4$

$\frac{y-4}{2} = (x-1)^2$

$1 \pm \sqrt{\frac{y-4}{2}} = x$

$1 - \sqrt{\frac{y-4}{2}} = x$



range h is $[6 +\infty)$

3. John invested \$ 1,000 in 2015. Mary invested \$ 1,000 in 2016.

   Assume both investments grow exponentially. John 's investment increases 3 % every two years. Mary's doubles every 15 years. When will Mary have three times as much money invested as John ? Give the answers in years (Ex: in the year 2040)

$t = 0$ corresponds to 2015

$f(t) = 1000 \cdot \left(\sqrt{1.03}\right)^t$ value of John's investment

$g(t) = 1000 \left(\sqrt[15]{2}\right)^{t-1}$ value of Mary's investment

want: $g(t) = 3 f(t)$

$$\cancel{1000}\left(\sqrt[15]{2}\right)^{t-1} = 3 \cdot \cancel{1000}\left(\sqrt{1.03}\right)^t$$

$$\left(\frac{\sqrt[15]{2}}{\sqrt{1.03}}\right)^t = 3 \sqrt[15]{2}$$

$$t = \frac{\ln\left(3\sqrt[15]{2}\right)}{\ln\left(\frac{\sqrt[15]{2}}{\sqrt{1.03}}\right)} \approx 36$$

ln 2051