Your Name

Your Signature

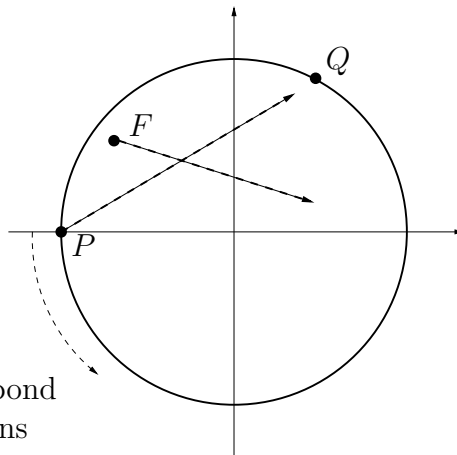Student ID #

Quiz Section

Professor's Name

TA's Name

- Give all answers with 2 decimal point accuracy.

- This exam is closed book. You may use one two-sided $8\frac{1}{2} \times 11''$ sheet of notes.

- Graphing calculators are not allowed. Do not share calculators.

- You must show your work on all problems.

- **To receive credit, you must write your final answer to each question in the box provided.**

- If you use a trial and error (or guess and check) method when an algebraic method is available, you will not receive full credit.

- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

- Raise your hand if you have a question.

Problem	Total Points	Score
1	21	
2	16	
3	10	
4	11	
5	9	

Problem	Total Points	Score
6	10	
7	15	
8	8	
Total	100	

1. [**21 points**] Artis and Betty are at the point P, on one side of a circular pond of radius 40 feet. Artis wades across the pond to the point Q, and Betty walks counter-clockwise around the edge of the pond.

PSfrag replacements

(a) [**3 points**] Artis wades across the pond according to the parametric equations

$$x(t) = 2t - 40$$
$$y(t) = t,$$

where x and y are given in feet and t in seconds. How fast, in feet per second, is Artis wading?

Answer:

(b) [**4 points**] Find the coordinates of the point Q.

Answer:

(c) [**3 points**] How far does Artis wade? That is, find the distance from P to Q.

Answer:

(d) [**4 points**] There is a fish at the point $F = (-30, 20)$ at $t = 0$, when Artis and Betty leave P. The movement of Artis and Betty startles the fish, so it swims away. Five seconds later, the fish is at the point $(30, 0)$. Assume the fish swims in a straight line and a constant speed. Write down parametric equations for the position of the fish after t seconds.

Answer:

(e) [**3 points**] Betty calculates that her angular speed is 8 radians per *minute*. Where is Betty 10 seconds after she leaves the point P?

Answer:

(f) [**4 points**] How long, in seconds, does it take Betty to walk from P to Q?

Answer:

2. [**16 points**] Clarence and Doris each make investments today.

(a) [**4 points**] Clarence deposits some money into a bank account that pays 4% annual interest, compounded continuously. There will be $5,500 in the account after 17 months. How much did Clarence deposit?

Answer:

(b) [**4 points**] In how many years will Clarence's account be worth 40% more than his initial deposit?

Answer:

(c) [**3 points**] Doris invests her $8,000 into a certificate of deposit that offers 5% annual interest, compounded monthly. Find a formula for the value of Doris's investment t years from now.

Answer:

(d) [**5 points**] When will Doris's investment be worth precisely twice the value of Clarence's investment?
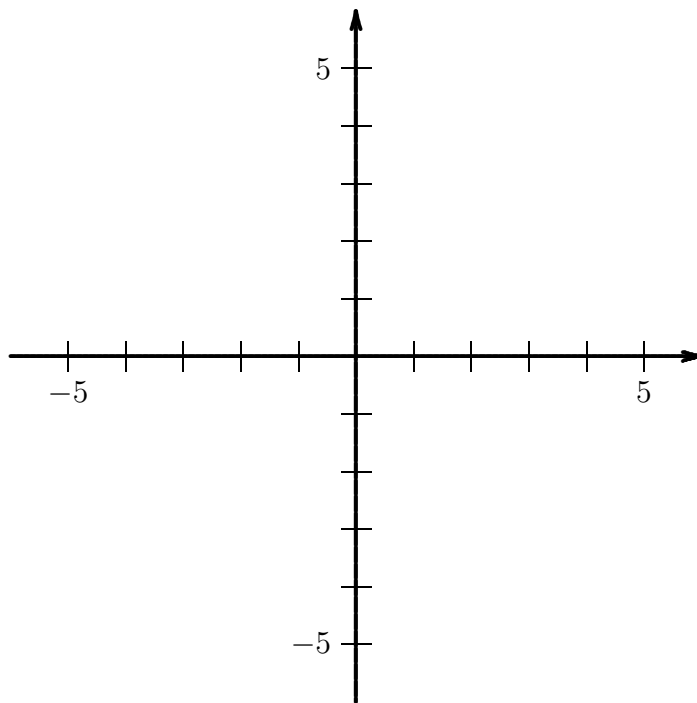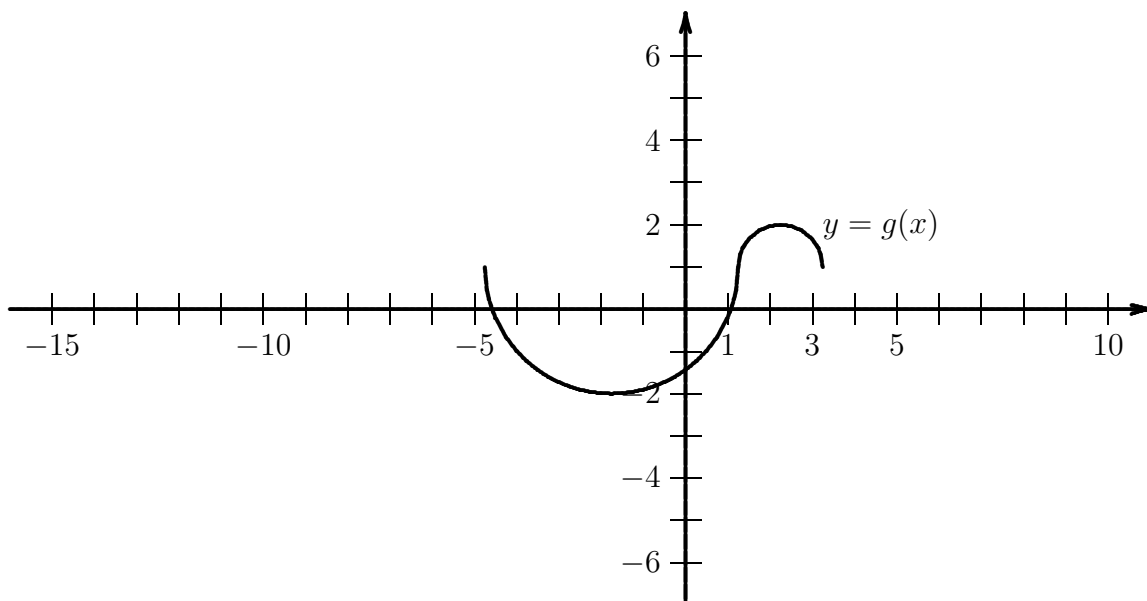
Answer:

3. [10 points]

(a) [**5 points**] Graph the function

$$y = f(x) = \begin{cases} 2x + 3 & \text{if } -4 \le x \le 0 \\ x^2 - 4x + 3 & \text{if } 0 < x \le 4 \end{cases}$$
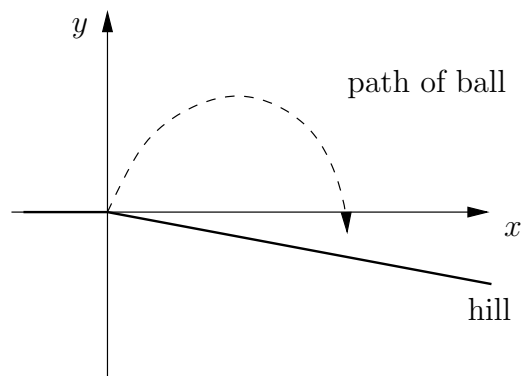
on the coordinate axes below.

(b) [**5 points**] The graph below is of $y = g(x)$. This graph consists of two semi-circles. On the same axes, graph $y = \dfrac{1}{2}g\left(\dfrac{1}{3}(x - 1)\right)$.

4. [**11 points**] Mia kicks a ball down a hill. As shown in the picture to the right, the ball follows the path of a parabola, namely

$$y = -\frac{1}{50}x^2 + \frac{8}{5}x$$

PSfrag replacements

in the given coordinate system. (Both x and y are given in units of feet.)

y

path of ball

x

hill

(a) [**3 points**] The hill drops 1 vertical foot for every 5 horizontal feet. Find the equation of the line that models the hill.

Answer:

(b) [**4 points**] Find the greatest vertical height of the ball over the hill.

Answer:

(c) [**4 points**] Find the coordinates of the point where the ball lands.

Answer:

5. [**9 points**] Cold water is poured into a glass in a warm room. The temperature of the water (in degrees Fahrenheit) t minutes after it was poured is given by

$$T(t) = 80 - 40e^{-0.02t}.$$

(a) [**3 points**] Find the temperature of the water 10 minutes after it was poured.

Answer:

(b) [**6 points**] When will the water temperature be 70° F? Give your answer in terms of the time t since the glass was poured.
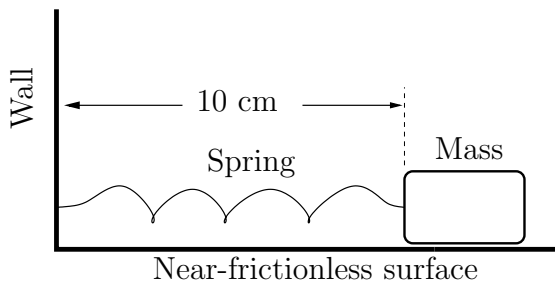
Answer:

6. [10 points]

(a) **[5 points]** Let $f(x) = \dfrac{3}{x} + \dfrac{x}{2}$. Simplify $\dfrac{f(x+h) - f(x)}{h}$ as much as possible.

Answer:

(b) **[5 points]** Let $f(x) = \dfrac{3}{x} + \dfrac{x}{2}$ as in part (a). Find all values of x for which $f(x) = 3$.

Answer:

7. [**15 points**]　You are performing a physics experiment with a mass, spring, and a near-frictionless surface.　You pull back the mass 10 cm from a wall (as in the picture) and release.　The mass then oscillates so that its distance from the wall can be modeled by a sinusoidal function. The closest the mass gets to the wall is 4 cm.

PSfrag replacements

Wall

|← —— 10 cm —— →|

Spring

Mass

Near-frictionless surface

(a) [**2 points**]　You observe that the mass returns to its starting point 6 times in the first 3 seconds. Find the period of the motion of the mass.

Answer:

(b) [**4 points**]　Model the distance of the mass from the wall as a sinusoidal function

$$d(t) = A \sin\left(\frac{2\pi}{B}(t - C) \right) + D$$

where t is the time in seconds since you released the mass, and d is in cm.　Find the values of A, B, C, and D.

$A =$

$B =$

$C =$

$D =$

(c) [**3 points**] How far from the wall is the mass 1/10 of a second after you released it?

Answer:

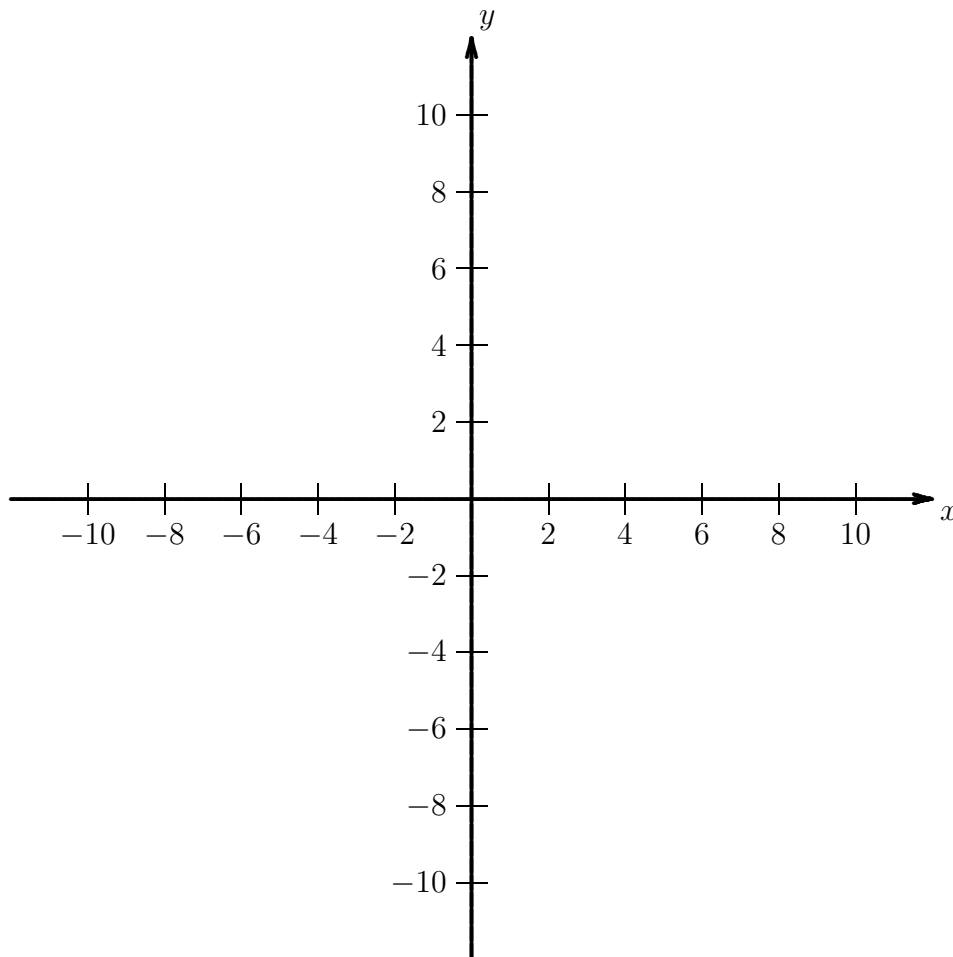(d) [**6 points**] Another mass has distance from the wall given by the function

$$D(t) = 4\sin\left(\frac{2\pi}{1/3}(t+1)\right) + 5.$$

Find the **first** two times when this mass is precisely 8 cm from the wall.

Answer:

8. [8 points] Let $f(x) = \dfrac{6 - 3x}{x + 3}$ for this problem.

(a) **[5 points]** Graph the function $y = f(x) = \dfrac{6 - 3x}{x + 3}$ on the axes, below. Clearly label at least four points on your graph, including all points where the curve crosses either axis. Be sure to show, and label, all asymptotes.

(b) **[3 points]** Recall that $f(x) = \dfrac{6 - 3x}{x + 3}$. Find the domain and range of the inverse function $y = f^{-1}(x)$. (You are not required to find the inverse function, only the domain and range.)

Answer: