1. (a) The general form of linear parametric equations is

$$v_x = x_0 + v_x t$$
$$v_y = y_0 + v_y t,$$

where (x_0, y_0) is the position at $t = 0$ and v_x and v_y are the horizontal and vertical velocities. From the equation given, we have $v_x = 2$ feet per second and $v_y = 1$ feet per second. Thus Artis's speed is $s = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1^2} = \sqrt{5} \approx 2.24$ feet per second.

(b) We wish to find Q, a point on the intersection of Artis's line of travel and the edge of the pond. This edge is a circle, centered at the origin, of radius 40 feet, and so it has equation $x^2 + y^2 = 40^2$. Artis's line of travel can be found by replacing t with y in $x = 2t - 40$ (since $y = t$); we get $x = 2y - 40$ or $y = x/2 + 20$. If we replace the x in the circle equation with $2y - 40$, we get

$$(2y - 40)^2 + y^2 = 40^2$$

or

$$5y^2 - 160y = 0.$$

There are two solutions: $y = 0$ (the point P) and $y = 32$. From this we find $x = 2(32) - 40 = 24$, so the point Q has coordinates $(24, 32)$.

(c) Artis wades the distance from $P = (-40, 0)$ to $Q = (24, 32)$. This distance is given by $d = \sqrt{(-40 - 24)^2 + (0 - 32)^2} = \sqrt{5120} = 32\sqrt{5}$ feet, or roughly 71.55 feet.

(d) As in part (a), we know that the general form of linear parametric equations is

$$v_x = x_0 + v_x t$$
$$v_y = y_0 + v_y t,$$

where (x_0, y_0) is the position at $t = 0$ and v_x and v_y are the horizontal and vertical velocities. We are told that $(x_0, y_0) = (-30, 20)$ is the position of the fish at $t = 0$, and that the fish is at the point $(30, 0)$ at $t = 5$ seconds. We use this to compute the velocities:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{30 - (-30) \text{ feet}}{5 - 0 \text{ seconds}} = 12 \text{ ft/s}$$
$$v_y = \frac{\Delta y}{\Delta t} = \frac{0 - 20 \text{ feet}}{5 - 0 \text{ seconds}} = -4 \text{ ft/s}.$$

Thus the parametric equations for the position of the fish are

$$x(t) = -30 + 12t$$
$$y(t) = 20 - 4t,$$

with t given in seconds and $x(t)$ and $y(t)$ in feet.

(e) A straightforward way to solve this problem is to recall that the parametric equations for Betty's position t seconds after she leaves the point P are given by

$$x(t) = r\cos(\theta_0 + \omega t)$$
$$y(t) = r\sin(\theta_0 + \omega t),$$

where $r = 40$ feet is the radius of the circle, $\omega = 8$ rads/min is the angular speed, $\theta_0 = \pi$ radians is Betty's angle (in standard position) at $t = 0$, and $t = 10$ seconds is the time. We simply need to be careful with the time units (we have both radians per *minute* and time in *seconds*): $\omega = 8/60$ radians per second. Thus we have

$$\begin{aligned} x &= 40\cos\left(\pi + \frac{8}{60} \cdot 10\right) \\ &= 40\cos\left(\pi + 4/3\right) \end{aligned}$$

and

$$y = 40\sin\left(\pi + 4/3\right),$$

or $(x, y) \approx (-9.41, -38.88)$.

(f) To find how long it takes Betty to get from P to Q, we need to find the angle θ from P to Q (counter-clockwise). This is π plus the angle from the positive x-axis to $Q = (24, 32)$. We can find the angle (call it ϕ) from the positive x-axis to $Q = (24, 32)$ by any number of trigonometric functions: $\tan(\phi) = 32/24$ or $\sin(\phi) = 32/40$ or $\cos(\phi) = 24/40$. Using tangent, we see that $\theta = \pi + \phi = \pi + \tan^{-1}(32/24)$.

Now we must use the fact that $\theta = \omega t$ and $\omega = 8$ radians per minute. Thus the time it takes Betty to walk from P to Q is

$$t = \frac{\theta}{\omega} = \frac{\pi + \tan^{-1}(32/24) \text{ radians}}{8 \text{ rads/min}} \approx 0.508611 \text{ mins} \approx 30.52 \text{ seconds}.$$

2. (a) Clarence's investment is compounded continuously, so it's value is modeled by the equation

$$A(t) = Pe^{rt},$$

where r is the annual interest rate (as a decimal), t is the time (in years), P is the initial account balance (the principal), and $A(t)$ is the account balance after t years. We are told that $r = 0.04$ and the account balance after 17 months (or $t = 17/12$ years) is \$5,500. To find P, we plug in this information:

$$\$5,500 = Pe^{0.04 \cdot 17/12}.$$

We solve and get $P = 5,500e^{-0.04 \cdot 17/12} \approx \$5,197.00$.

(b) Clarence's initial deposit is P, his principal. (We found P in part (a), but the actual number isn't needed for this problem.) When his account is worth 40% more than P, it is worth $1.4P$. Thus we want to find t so that $A(t) = 1.4P$, or $1.4P = Pe^{0.04t}$. The two Ps cancel, and we're left with $e^{0.04t} = 1.4$. Taking natural logarithms, we get $\ln(1.4) = 0.04t$, or $t = \ln(1.4)/0.04 \approx 8.41$ years.

(c) Doris's investment is compounded monthly, so its value is given by the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt},$$

where $A(t)$, $P = \$8,000$, $r = 0.05$, and t all mean the same thing as in Clarence's investment (although, of course, they will have different values), and $n = 12$ is the number of compounds per year. Thus Doris's investment is worth

$$A(t) = \$8,000\left(1 + \frac{0.05}{12}\right)^{12t},$$

in t years.

(d) We wish to find when Doris's investment will be worth twice that of Clarence. Writing $A_D(t)$ for the value of Doris's investment and $A_C(t)$ for Clarence's, we want to solve $A_D(t) = 2A_C(t)$ for t. Written out, this equation is

$$\$8,000\left(1 + \frac{0.05}{12}\right)^{12t} = 2\left(\$5,197e^{0.04t}\right).$$

Taking the natural logarithm of both sides, we get

$$\ln\left(8,000\left(1 + \frac{0.05}{12}\right)^{12t}\right) = \ln\left(10394e^{0.04t}\right)$$

or, simplifying using the rule $\ln(ab) = \ln(a) + \ln(b)$,

$$\ln(8,000) + \ln\left(\left(1 + \frac{0.05}{12}\right)^{12t}\right) = \ln(10394) + \ln(e^{0.04t})$$
$$\ln(8,000) + 12t\ln\left(1 + \frac{0.05}{12}\right) = \ln(10394) + 0.04t.$$

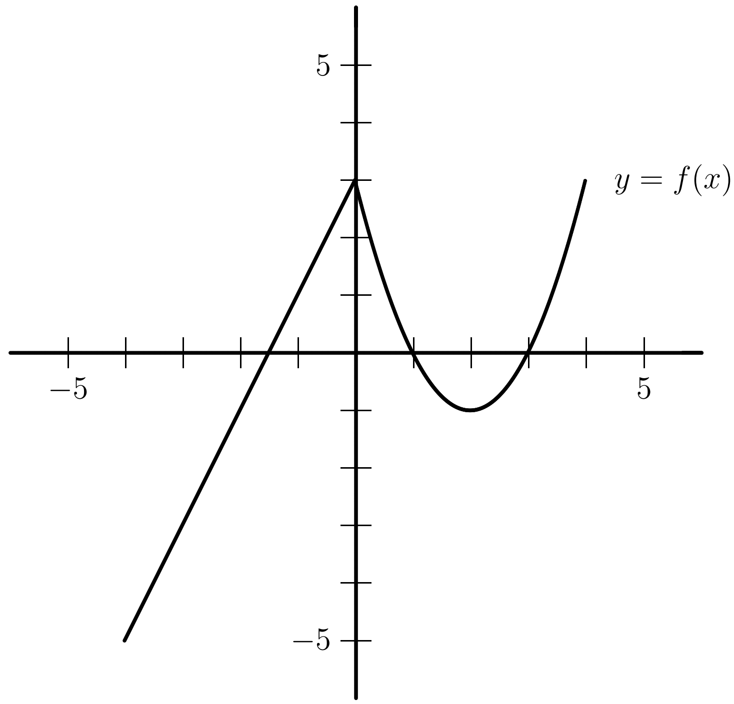
Grouping all the terms with t on the left, and all the other terms on the right, we get

$$\left(12\ln\left(1 + \frac{0.05}{12}\right) - 0.04\right)t = \ln(10394) - \ln(8000)$$

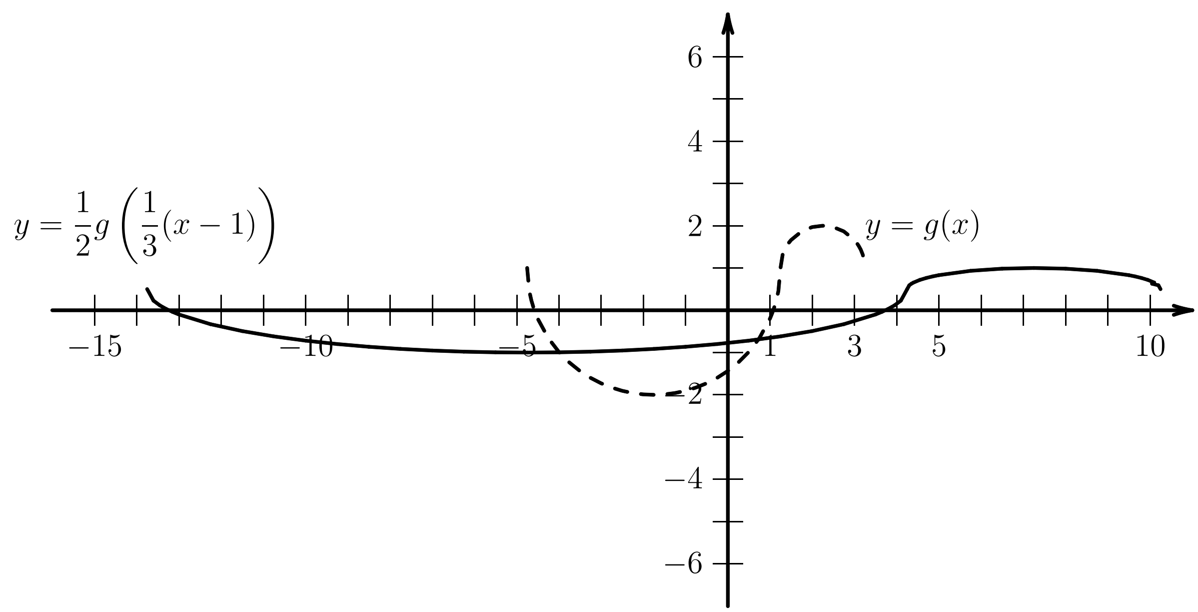
or

$$t = \frac{\ln(10394) - \ln(8000)}{12\ln\left(1 + \frac{0.05}{12}\right) - 0.04} \approx 26.45 \text{ years.}$$

3. (a) The function $y = f(x)$ is a straight line segment connecting the points $(-4, -5)$ and $(0, 3)$ together with the parabola $y = x^2 - 4x + 3$ on the domain $0 < x \leq 4$. This parabola opens up, has vertex at $(2, -1)$, and passes through the points $(0, 3)$ and $(4, 3)$. The graph of $y = f(x)$ is shown below:



(b) We show on the axes below both the graph of $y = g(x)$ (now dotted) and the graph of $y = \frac{1}{2}g\left(\frac{1}{3}(x-1)\right)$ (the solid curve). The 1/2 has compressed the graph vertically, the 1/3 has stretched it horizontally, and the 1 has shifted it one unit to the right.



4. (a) The line of the hill passes through the origin, so the y-intercept is $b = 0$. The description "drops 1 vertical foot for every 5 horizontal feet" means that when $\Delta x = 5$, we have $\Delta y = -1$. Thus the slope of this line is $m = \frac{\Delta y}{\Delta x} = \frac{-1}{5}$. Hence the equation $y = mx + b$ is $y = -\frac{1}{5}x$.

(b) The vertical height of the ball over the hill is given by taking the difference in the ball's and hill's y-coordinates:

$$\begin{aligned}\text{vertical height over hill} &= y_{\text{ball}} - y_{\text{hill}} \\ &= \left(-\frac{1}{50}x^2 + \frac{8}{5}x\right) - \left(-\frac{1}{5}x\right) \\ &= -\frac{1}{50}x^2 + \frac{9}{5}x.\end{aligned}$$

This vertical height is greatest at the vertex:

$$x = -\frac{b}{2a} = -\frac{9/5}{2(-1/50)} = 45.$$

When $x = 45$, the vertical height of the ball over the hill is given by $-\frac{1}{50}(45)^2 + \frac{9}{5}(45) = 40.5$ feet.

(c) The ball lands where the parabola that models the path of the ball intersects the line that models the hill. That is, the ball lands where $y_{\text{ball}} = y_{\text{hill}}$. In terms of x, this equation is

$$-\frac{1}{50}x^2 + \frac{8}{5}x = -\frac{1}{5}x^2.$$

We can rewrite this quadratic equation as $-\frac{1}{50}x^2 + \frac{9}{5}x = 0$ and solve; this has solutions $x = 0$ (the point where the ball is kicked) and $x = 90$ (where the ball lands). The y-coordinate of this point is found most easily by plugging $x = 90$ into the equation for the line: $y = -\frac{1}{5}(90) = -18$. Thus the ball lands at $(x, y) = (90, -18)$.

5. (a) The question asks for the temperature at $t = 10$; this is simply $T(10) = 80 - 40e^{-0.02(10)} \approx 47.25°$ Fahrenheit.

(b) Now we wish to find the time t when $T(t) = 70$. That is, we wish to solve $80 - 40e^{-0.02t} = 70$. This is equivalent to $e^{-0.02t} = 1/4$. By taking natural logarithms, we get $-0.02t = \ln(1/4)$, or $t = -50\ln(1/4) \approx 69.31$ minutes.

6. (a) Recall that $f(x) = \frac{3}{x} + \frac{x}{2}$. Then, replacing x with $x + h$, we get $f(x+h) = \frac{3}{x+h} + \frac{x+h}{2}$. Hence we have

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{1}{h}\left[\frac{3}{x+h} + \frac{x+h}{2} - \left(\frac{3}{x} + \frac{x}{2}\right)\right] \\ &= \frac{1}{h}\left(\frac{3}{x+h} - \frac{3}{x} + \frac{x+h}{2} - \frac{x}{2}\right) \\ &= \frac{1}{h}\left(\frac{3x}{x(x+h)} - \frac{3(x+h)}{x(x+h)} + \frac{x+h-x}{2}\right) \\ &= \frac{1}{h}\left(\frac{3x - 3x - 3h}{x(x+h)} + \frac{x+h-x}{2}\right) \\ &= \frac{1}{h}\left(\frac{-3h}{x(x+h)} + \frac{h}{2}\right).\end{aligned}$$

Dividing through by h, we get $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-3}{x(x+h)} + \dfrac{1}{2}$, or $\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + xh - 6}{2x(x+h)}$.

(b) To find where $f(x) = 3$, we simply set $f(x) = \frac{3}{x} + \frac{x}{2} = 3$. Multiplying through by $2x$ to clear the denominators, we have $6 + x^2 = 6x$, or $x^2 - 6x + 6 = 0$. The quadratic formula says that this has roots when

$$x = \frac{-6 \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)} = \frac{6 \pm \sqrt{12}}{2} = 3 \pm \sqrt{3},$$

or when $x \approx 4.73$ and $x \approx 1.27$.

7. (a) The phrase "the mass returns to its starting point 6 times in the first 3 seconds" means that 6 periods take 3 seconds. Thus 1 period takes $3/6 = 1/2$ seconds.

(b) From part (a), we have $B = 1/2$ seconds. We are also told that the maximum distance from the wall (which occurs at time $t = 0$ seconds) is 10 cm, and the minimum distance is 4 cm. Thus $D = \frac{\max+\min}{2} = 7$ cm, $A = \max - D = 3$ cm (or $A = D - \min$ or $A = \frac{\max-\min}{2}$), and $C = t_{\max} - \frac{1}{4}B = 0 - \frac{1}{4}(1/2) = -1/8$ seconds. Thus the model is

$$d(t) = 3\sin\left(\frac{2\pi}{1/2}(t + 1/8)\right) + 7.$$

(c) This question asks for $d(1/10)$, the distance the mass is from the wall at time $t = 1/10$ seconds. We simply plug in to the formula we just produced to get

$$d(1/10) = 3\sin\left(\frac{2\pi}{1/2}(1/10 + 1/8)\right) + 7 \approx 7.93 \text{ cm}.$$

(d) Now we are asked to find the first two times when $D(t) = 8$. Note that this is a new function given to us for this part. This amounts to solving the equation

$$\sin\left(\frac{2\pi}{1/3}(t + 1)\right) = \frac{3}{4}$$

for t. The principal solution is simply from

$$\frac{2\pi}{1/3}(t + 1) = \sin^{-1}(3/4),$$

or $t = -1 + \frac{1}{6\pi}\sin^{-1}(3/4) \approx -0.955$ seconds.
The symmetry solution is found from

$$\frac{2\pi}{1/3}(t + 1) = \pi - \sin^{-1}(3/4),$$

or $t = -1 + \frac{1}{6\pi}\left(\pi - \sin^{-1}(3/4)\right) \approx -0.878$ seconds.
To find the *first* two solutions, we need to add multiples of the period $B = 1/3$ seconds to each of these answers until we first get positive values:

$$t = -1 + \frac{1}{6\pi}\sin^{-1}(3/4) + 3B \approx -0.955 + 3(1/3) \approx 0.045 \text{ seconds}$$

and

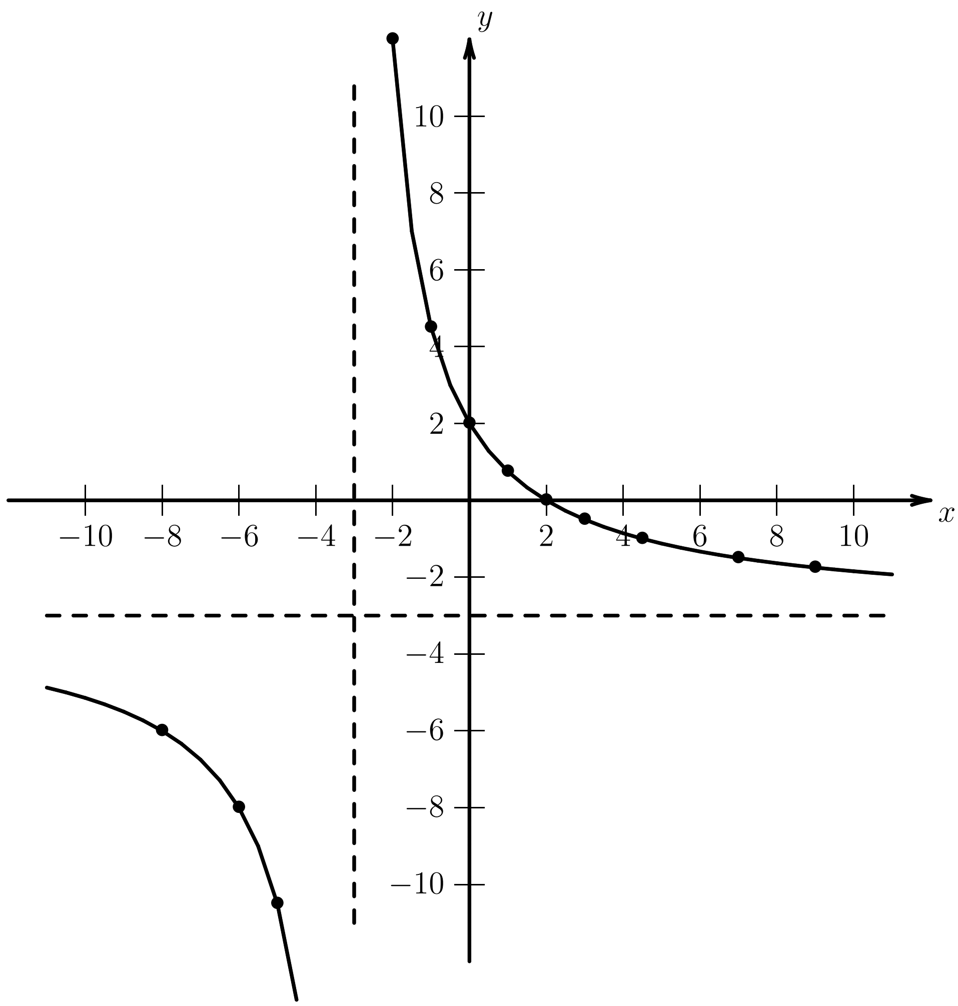
$$t = -1 + \frac{1}{6\pi}\left(\pi - \sin^{-1}(3/4)\right) + 3B \approx -0.878 + 3(1/3) \approx 0.122 \text{ seconds}.$$

These are the first two times that this mass is precisely 8 cm from the wall.

8. (a) The graph of $y = \dfrac{6-3x}{x+3}$ is shown below. The graph crosses the x axis when $y = 0$, which occurs when $6 - 3x = 0$, or $x = 2$. The graph crosses the y axis when $x = 0$, in which case $y = \frac{6-0}{0+3} = 2$. The vertical asymptote is where the denominator $x + 3$ is zero; this happens at $x = -3$. We find the horizontal asymptote by multiplying the top and bottom by $1/x$ and letting x get large:

$$y = \frac{6-3x}{x+3} \cdot \frac{1/x}{1/x} = \frac{6/x - 3}{1 + 3/x} \approx \frac{0-3}{1+0} = -3.$$

When x gets large, both $6/x$ and $3/x$ are roughly zero, so y is roughly -3. Thus the horizontal asymptote is $y = -3$. Now we plot points and get the following graph. I have specifically filled in the following points on the graph: $(-8, -6)$, $(-6, -8)$, $(-5, -10.5)$, $(-2, 12)$, $(-1, 4.5)$, $(0, 2)$, $(1, 3/4)$, $(2, 0)$, $(3, -1/2)$, $(4.5, -1)$, $(7, -3/2)$, and $(9, -7/4)$.



(b) The domain of $x = f^{-1}(y)$ is the range of $y = f(x)$, or all $x \neq -3$. Similarly, the range of $x = f^{-1}(y)$ is the domain of $y = f(x)$, or all $y \neq -3$. Writing these as the domain and range of $y = f^{-1}(x)$ (that is, switching the letters), we have

The domain of $y = f^{-1}(x)$ is $\{$all real x with $x \neq -3\}$

The range of $y = f^{-1}(x)$ is $\{$all real y with $y \neq -3.\}$