$\boxed{1}$ (a) The parachutist's speed is 15 feet per second, so after t seconds he has gone $15t$ feet. He is traveling down, with a height at $t = 0$ of 153, so his height at time t seconds is $h(t) = 153 - 15t$ feet.

(b) The baseball that Ichiro throws is highest at the vertex of the parabola $h = -16t^2 + 91t + 5$. This occurs at

$$t = -\frac{b}{2a} = -\frac{91}{2(-16)} = \frac{91}{32} \text{ seconds,}$$

or $t = 2.84375$ seconds.

(c) For this problem, we wish to find the *second* time the parachutist and the ball have the same height. This occurs when the two height equations are equal:

$$-16t^2 + 91t + 5 = 153 - 15t,$$

or $0 = 16t^2 - 106t + 148$. If we can't factor this, we can always use the quadratic formula to produce the two solutions:

$$\begin{aligned} t &= \frac{+106 \pm \sqrt{(-106)^2 - 4(16)(148)}}{2(16)} \\ &= \frac{106 \pm \sqrt{1764}}{32} \\ &= \frac{106 \pm 42}{32} \\ &= 2 \text{ seconds or } \ 4.625 \text{ seconds.} \end{aligned}$$

The second time is thus $t = 4.625$ seconds.

(d) The distance that the ball is above the parachutist is given by

$$\begin{aligned} d &= h_{\text{ball}} - h_{\text{parachutist}} \\ &= \left(-16t^2 + 91t + 5\right) - (153 - 15t) \\ &= -16t^2 + 106t - 148. \end{aligned}$$

This distance is *greatest* at the vertex (this is a parabola that opens downward), so the distance is greatest at

$$t = -\frac{b}{2a} = -\frac{106}{2(-16)} = \frac{53}{16} = 3.3125 \text{ seconds.}$$

This is the time when the distance is greatest, but we are asked for the distance. We plug back in to the distance formula, and we get:

$$\begin{aligned} d &= -16\left(\frac{53}{16}\right)^2 + 106\left(\frac{53}{16}\right) - 148 \\ &= \frac{441}{16} = 27.5625 \text{ feet.} \end{aligned}$$

This is the greatest distance that the ball is above the parachutist.

$\boxed{2}$ (a) Where the ball exits the green is the intersection of the line giving the path of the ball and the circle outlining the green. In terms of the given coordinate system, we can find the equations of this line and this circle.

First, the line. The ball starts at $(-5,-40)$ and passes onto the green at $(0,-30)$, so the line has slope

$$m = \frac{-30 - -40}{0 - -5} = \frac{10}{5} = 2$$

We could plug in a point to $y = 2x + b$ to find b, but it is simpler to notice that the point $(0,-30)$ tells us that the y-intercept is $b = -30$. Thus the line has equation $y = 2x - 30$.

The circle has center at the origin and radius 30 feet, so its equation is $(x-0)^2 + (y-0)^2 = 30^2$, or $x^2 + y^2 = 900$.

To find the intersection, we replace the y in the circle with $2x - 30$ to get

$$x^2 + (2x - 30)^2 = 900,$$

or

$$x^2 + 4x^2 - 120x + 900 = 900.$$

This equation simplifies to $5x^2 - 120x = 0$, or $5x(x - 24) = 0$. This has roots at $x = 0$ (when the ball enters the green) and $x = 24$ (where the ball exits the green). We want the latter point. To find y, we plug $x = 24$ into the circle or (easier) the line: $y = 2(24) - 30 = 18$. Thus the point the ball exits the green is $(x, y) = (24, 18)$.

(b) This question asks for the time the ball takes to go from $(-5,-40)$ to $(24, 18)$, traveling at a constant rate of 10 feet per second. We first need to compute the distance traveled, then the time (using distance equals rate times time). The distance traveled is computed using the distance formula:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-5 - 24)^2 + (-40 - 18)^2} = \sqrt{(-29)^2 + (-58)^2} = 29\sqrt{5} \text{ feet.}$$

The time is distance divided by speed (or rate), so

$$t = \frac{\text{distance}}{\text{speed}} = \frac{29\sqrt{5} \text{ feet}}{10 \text{ ft per sec}} = \frac{29\sqrt{5}}{10} \text{ seconds,}$$

or roughly 6.48 seconds.

(c) The ball is closest to the cup when the line of the ball's travel intersects a perpendicular line through the cup. The line of the ball's path has slope $m = 2$, so the perpendicular line has slope $-1/2$. It passes through $(5, 0)$ (the cup), so this perpendicular line is $y - 0 = -\frac{1}{2}(x - 0)$, or $y = -\frac{1}{2}x + \frac{5}{2}$.

The perpendicular line $y = -\frac{1}{2}x + \frac{5}{2}$ intersects the line of the ball's path $y = 2x - 30$ when $-\frac{1}{2}x + \frac{5}{2} = 2x - 30$. This means $2.5x = 32.5$, or $x = 13$. Plugging in to either equation, we find $(x, y) = (13, -4)$ is the closest point on the ball's path to the cup.

$\boxed{3}$ (a) Recall that $f(x) = x^2 + 1$, so $f(x+h) = (x+h)^2 + 1$. We can simplify:

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^2 + 1 - \left(x^2+1\right)}{h} \\ &= \frac{x^2 + 2hx + h^2 + 1 - x^2 - 1}{h} \\ &= \frac{2hx + h^2}{h} \\ &= 2x + h.\end{aligned}$$

This is our simplified expression.

(b) Recall that $g(x) = \dfrac{x}{x+1}$ and $h(x) = \dfrac{1-x}{2x}$. We want to find the expression for $g(h(x))$ as well as the domain of this composition.

First, the rule is

$$g(h(x)) = g\left(\frac{1-x}{2x}\right) = \frac{\frac{1-x}{2x}}{\frac{1-x}{2x}+1}.$$

We multiply the numerator and denominator by $2x$ to simplify this expression to

$$g(h(x)) = \frac{1-x}{1-x+2x} = \frac{1-x}{1+x}.$$

The domain of this function is

$$\{x \text{ in the domain of } h : h(x) \text{ is in the domain of } g\}.$$

The domain of h is all real numbers except 0. The domain of g is all real numbers except -1, so we need to find x so that $h(x) = -1$. That is, what is x if $\frac{1-x}{2x} = -1$. We multiply by $2x$ to get $1 - x = -2x$, then solve: $x = -1$. Thus the domain of $g(h(x))$ is the set of all real numbers except $x = 0$ (since 0 is not in the domain of h) and $x = -1$ (since $h(-1)$ is not in the domain of g).

$\boxed{4}$ (a) The graph for $y = f(x)$ has two parts: a straight line on the domain $-4 \le x \le 0$, and a quarter circle on the domain $0 < x \le 4$. The line connects $(-4, 3)$ and $(0, -1)$, so it has slope $m = \frac{3--1}{-4-0} = -1$, and y-intercept $b = -1$. Thus the equation of the line is $y = -x - 1$. The quarter circle is centered at $(x, y) = (0, 3)$ with radius $r = 4$, so the circle has equation $(x-0)^2 + (y-3)^2 = 4^2$. Solving for y, we get the two functions $y = 3 \pm \sqrt{16 - x^2}$. Since $y \le 3$, we want $y = 3 - \sqrt{16 - x^2}$. Thus our function is

$$f(x) = \begin{cases} -x - 1 & \text{if } -4 \le x \le 0 \\ 3 - \sqrt{16 - x^2} & \text{if } -4 \le x \le 0. \end{cases}$$

(b) The graph of $y = 3f(x+1)$ appears on the axes below. The dotted graph is the graph of the original function $y = f(x)$. The "3" produces a vertical dilation (a "stretch") by a factor of 3. The "+1" gives a horizontal shift of 1 to the left.

EXAM ONE SOLUTIONS