Math 120A **Exam Two Solutions** Spring, 2002

**1** (a) One way to find the amplitude $A$ is simply to take the maximum minus the mean. The mean here is $D = 60$ (we're told this), and the maximum is 100. This means the amplitude is $A = 100 - 60 = 40$ gallons.

(b) We've seen in the solution to part (a) that $A = 40$ and $D = 60$. The period is one year, or $B = 365$ days. Finally, we're told that the time at the maximum is $t_{\text{max}} = 213$ days, so one possible phase shift is $C = t_{\text{max}} - B/4 = 121.75$ days. Thus our formula is

$$s(t) = 40 \sin\left(\frac{2\pi}{365}(t - 121.75)\right) + 60.$$

(c) Now we are asked to find $s(137)$, the amount of ice cream on day 137 (May 17th). This is simply

$$\begin{aligned} s(t) &= 40 \sin\left(\frac{2\pi}{365}(137 - 121.75)\right) + 60 \\ &\approx 70.38 \text{ gallons.} \end{aligned}$$

(d) Now we wish to solve $s(t) = 70$, or

$$70 = 40 \sin\left(\frac{2\pi}{365}(t - 121.75)\right) + 60,$$

or

$$\frac{1}{4} = \sin\left(\frac{2\pi}{365}(t - 121.75)\right).$$

Thus one (principal) solution is

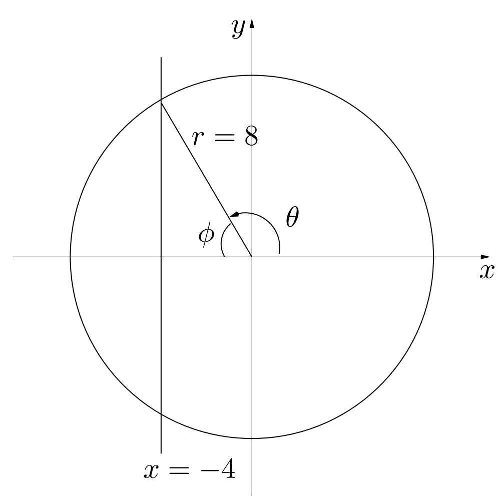
$$t = \frac{365}{2\pi} \sin^{-1}(1/4) + 121.75 \approx 136.43.$$

Another solution would be one period later: $t = 136.43 + 365 = 501.43$. Other solutions can be found using the symmetry solution:

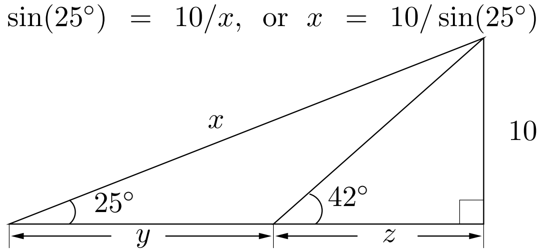
$$t = \frac{365}{2\pi}\left(\pi - \sin^{-1}(1/5)\right) + 121.75 \approx 289.57,$$

and from this we get yet another solution one period later: $t = 289.57 + 365 = 654.57$.

**2** (a) I've added the angle $\phi$ in the circle to the right. From the picture, we see that $\theta + \phi = \pi$. Moreover, the triangle involving $\phi$ has side adjacent to $\phi$ has length 4 and hypotenuse has length 8. Thus $\cos(\phi) = 4/8$, or $\phi = \pi/3$. Hence $\theta = \pi - \pi/3 = 2\pi/3$.




(b) To find the length $x$, we look at the larger right triangle. From this triangle we see that $\sin(25°) = 10/x$, or $x = 10/\sin(25°) \approx 23.66$.



(c) To find the length $y$, we have added the notation "$z$" in the picture. We then get two relations involving tangent from the two right triangles:

$$\tan(25°) = \frac{10}{y+z} \qquad \tan(42°) = \frac{10}{z}.$$

Solving, we get

$$y + z = \frac{10}{\tan(25°)} \qquad \text{and} \qquad z = \frac{10}{\tan(42°)},$$

or, solving for $y$,

$$y = \frac{10}{\tan(25°)} - \frac{10}{\tan(42°)} \approx 10.34.$$

3 (a) We are asked for the linear speed $v_B$ of wheel $B$. We find this using the formula $v = r\omega$ (which only applies if $\omega$ is measured in radians per time unit!). We're given $r_B = 2$ inches, and since wheels $A$ and $B$ are fastened at the axle, they have the same angular velocity. Thus

$$\omega_B = \omega_A = \left(200\ \frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi\ \text{rads}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = \frac{20\pi}{3}\ \text{rads/sec}.$$

Thus the linear speed of wheel $B$ is

$$v_B = (2\ \text{inches})\left(\frac{20\pi}{3}\ \text{rads/sec}\right) = \frac{40\pi}{3}\ \text{in/sec} \approx 41.89\ \text{in/sec}.$$

(b) Now we're asked for wheel $C$'s angular velocity. Using $v = r\omega$, we get $\omega = v/r$. Since wheels $B$ and $C$ are joined by a belt, they have the same linear speed: $v_C = v_B = 40\pi/3$ in/sec. Since $r_C$ is given as 4 inches, we get

$$\omega_C = \frac{v_C}{r_C} = \frac{40\pi/3\ \text{in/sec}}{4\ \text{in}} = 10\pi/3\ \text{rads/sec}.$$

We convert this angular speed to RPM as follows:

$$\omega_C = \left(\frac{10\pi\ \text{rads}}{3\ \text{sec}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rads}}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right) = 100\ \text{RPM}.$$

This is the form of the answer we want.

4 (a) The zero is when $6 - 3x = 0$, or $(x, y) = (2, 0)$. The vertical asymptote is when $x - 6 = 0$, or $x = 6$. The horizontal asymptote can be found by multiplying the top and bottom by $1/x$ and letting $x$ go off to infinity:

$$\frac{6-3x}{x-6} \cdot \frac{1/x}{1/x} = \frac{6/x - 3}{1 - 6/x} \longrightarrow \frac{0-3}{1-0} = -3.$$

Thus $y = -3$ is the horizontal asymptote.

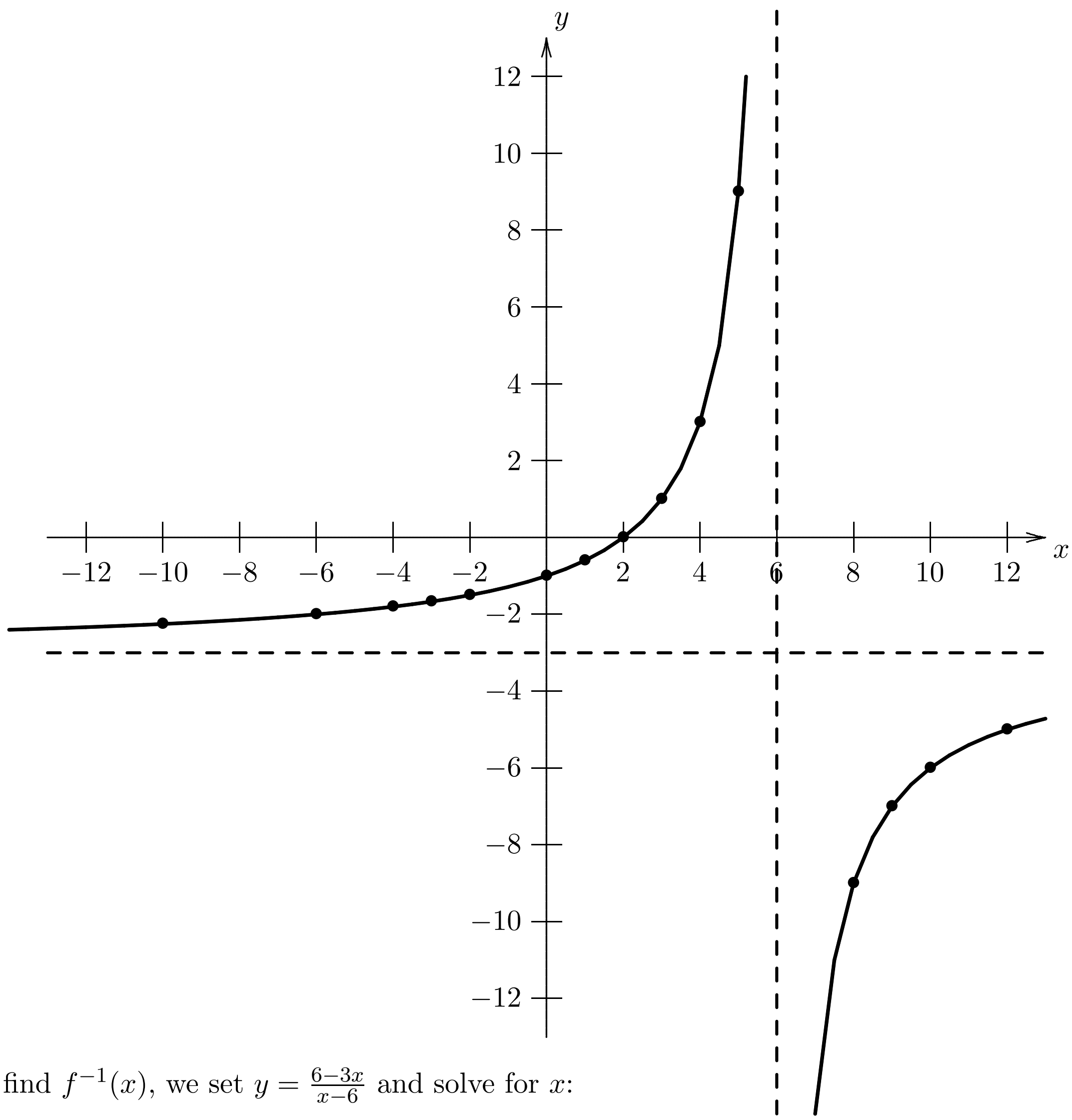
We plot the graph below, including many points (especially $(x, y) = (0, -1)$, where the curve crosses the $y$-axis). I've included dots at the following points:

$$(-10, -9/4) \quad (-6, -2) \quad (-4, -9/5) \quad (-3, -5/3) \quad (-2, -3/2)$$

$$(0, -1) \quad (2, 0) \quad (3, 1) \quad (4, 3) \quad (5, 9)$$

$$(8, -9) \quad (9, -7) \quad (10, -6) \quad (12, -5)$$

You could, of course, plot many points other than these.




(b) To find $f^{-1}(x)$, we set $y = \frac{6-3x}{x-6}$ and solve for $x$:

$$\begin{aligned} y(x-6) &= 6-3x \\ xy-6y &= 6-3x \\ x(y+3) &= 6+6y \\ x &= \frac{6+6y}{y+3}. \end{aligned}$$

Thus $f^{-1}(x) = \dfrac{6x+6}{x+3}$. The domain of this function is all $x \neq -3$ and the range is all $y \neq 6$. (These are the range and domain, in that order, of the original function $y = f(x)$.)