

Quiz Two Solutions

MATH 120A SPRING, 2002

- 1 The ball is highest above the x axis at the vertex: $x = -\frac{b}{2a}$. Here $a = -1/10$ and $b = 2$, so the x coordinate of the vertex is

$$x = -\frac{2}{2(-1/10)} = 10.$$

We find the y coordinate by simply plugging in $x = 10$; we get $y = -\frac{1}{10}(10)^2 + 2(10) + 20 = 30$. Thus the highest point above the x axis is $(x, y) = (10, 30)$.

- 2 The multi-part function consists of three lines: two horizontal lines, and a line connecting a point at $x = 5$ to the point $(25, 0)$. The first horizontal line, on the domain $-10 \leq x \leq 5$, is $y = c$ for some constant c . What is c ? It is the height of the ball at $x = 0$, or $y = -\frac{1}{10}(0)^2 + 2(0) + 20 = 20$. Thus the first part of the function is $y = 20$.

The non-horizontal line now connects the points $(5, 20)$ and $(25, 0)$. This line has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{20 - 0}{5 - 25} = \frac{20}{-20} = -1.$$

The equation of this line is thus $y - 20 = -1(x - 5)$ or $y = -x + 25$.

The final horizontal line is simply $y = 0$, so the full multi-part formula is

$$y = \begin{cases} 20 & \text{if } -10 \leq x \leq 5 \\ -x + 25 & \text{if } 5 < x < 25 \\ 0 & \text{if } 25 \leq x \leq 30. \end{cases}$$

- 3 The ball lands when $y = 0$, so we must solve the equation

$$-\frac{1}{10}x^2 + 2x + 20 = 0.$$

We use the quadratic formula with $a = -1/10$, $b = 2$, and $c = 20$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-1/10)20}}{2(-1/10)} = \frac{-2 \pm \sqrt{4 + 8}}{-1/5} \\ &= -5(-2 \pm \sqrt{4 + 8}) = -5(-2 \pm 2\sqrt{3}) = 10(1 \mp \sqrt{3}). \end{aligned}$$

There are two solutions to the equation, but only one correct answer to the problem: the ball lands at $x = 10(1 + \sqrt{3}) \approx 27.32$ feet.

- 4 For this problem, we must find the vertex of the equation for the height of the ball above the ground. Let us call this height $h(x)$; it is the difference of the height of the ball above the x axis, and the height of the ground above the x axis:

$$\begin{aligned} h(x) &= \left(-\frac{1}{10}x^2 + 2x + 20\right) - (-x + 25) \\ &= -\frac{1}{10}x^2 + 3x - 5. \end{aligned}$$

The vertex of this is at $x = -\frac{3}{2(-1/10)} = 15$.