Quiz Three Solutions

Math 120A Spring, 2002

1 Recall that $f(x) = 4 - x^2$ and $g(x) = x + 1$. To find all values of x for which $\dfrac{f(x)}{g(x)} = 4$, we simply replace the functions by their formulas and solve:

$$\frac{4 - x^2}{x + 1} = 4.$$

We multiply both sides of the equation by $x + 1$ to obtain

$$4 - x^2 = 4(x + 1) = 4x + 4.$$

This simpifies to $x^2 = -4x$, which has two solutions: $x = 0$ and $x = -4$.

2 To find an inverse function $y = f^{-1}(x)$ for the function $f(x) = 4 - x^2$, we need to restrict the domain of $f(x)$ to obtain a *one-to-one* function. We can restrict to $D_1 = \{x \leq 0\}$ (the vertex and left) or to $D_2 = \{x \geq 0\}$ (the vertex and right). In either case the range is $R = \{y \leq 4\}$.

Since the inverse function maps the original function's *range* to its *domain*, we should expect $y = f^{-1}(x)$ to have domain $\{x \leq 4\}$ and range either $\{y \leq 0\}$ or $\{y \geq 0\}$ (depending on which inverse we take).

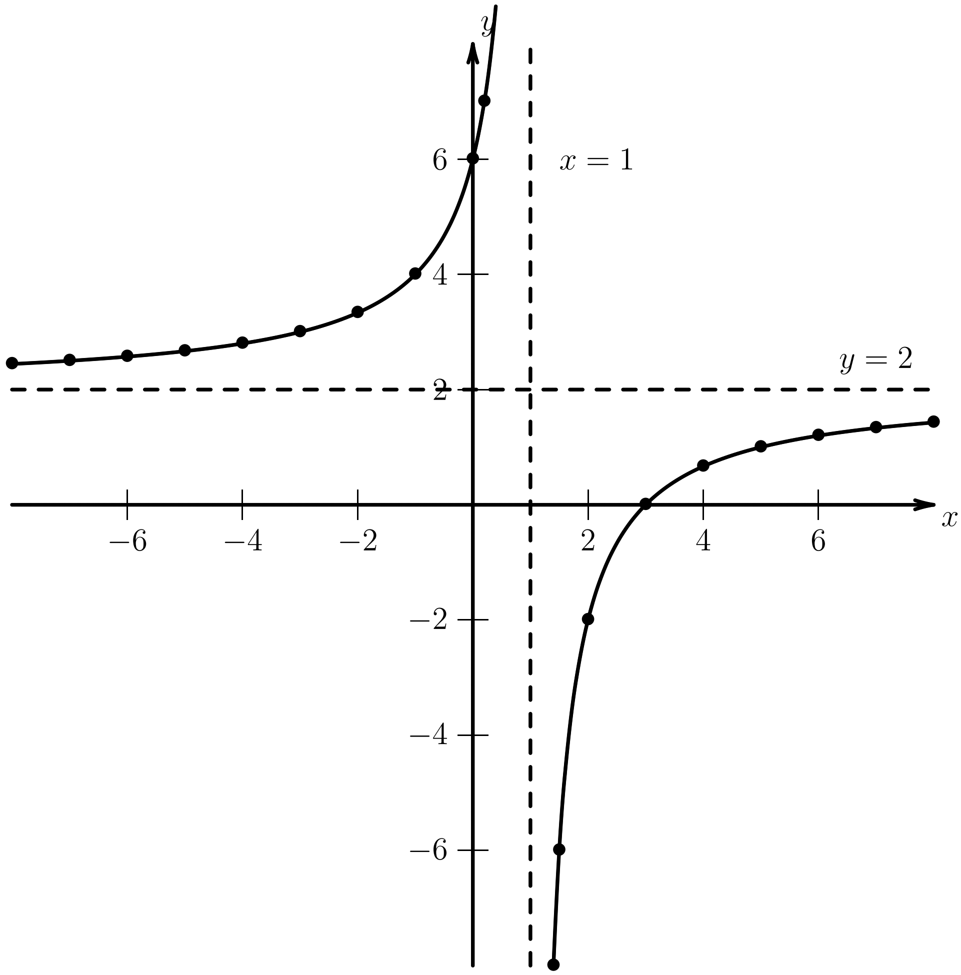
Let's now find the inverse: we want to solve $y = 4 - x^2$ for x. The simplest way is the direct way: $x^2 = 4 - y$, so $x = +\sqrt{4 - y}$ or $x = -\sqrt{4 - y}$. Thus our two two possible inverse functions are $f^{-1}(y) = +\sqrt{4 - y}$ or $f^{-1}(y) = -\sqrt{4 - y}$. Written in terms of x, and including the domains and ranges, we get two possible answers: $f^{-1}(x) = +\sqrt{4 - x}$ with domain $\{x \leq 4\}$ and range $\{y \geq 0\}$, or $f^{-1}(x) = -\sqrt{4 - x}$ with domain $\{x \leq 4\}$ and range $\{y \leq 0\}$.

3 We find the zeros and asymptotes of $h(x) = \dfrac{2x - 6}{x - 1}$ as follows. The zeros are where the numerator $2x - 6$ is zero; this occurs at $x = 3$. (This is the point $(x, y) = (3, 0)$.) The vertical asymptote is the line $x = 1$; we find this by solving for where the denominator of $h(x)$ is zero. Finally, to find the horizontal asymptote is found by muliplying the top and bottom of $h(x)$ by $1/x$ as follows:

$$h(x) = \frac{2x - 6}{x - 1} \cdot \frac{1/x}{1/x} = \frac{2 - \frac{6}{x}}{1 - \frac{1}{x}}.$$

As x gets large (in a positive or negative direction), $1/x$ and $6/x$ get close to zero, and so $h(x)$ gets close to $2/1 = 2$. Thus the horizontal asymptote is $y = 2$.

4 Here is the graph of the function $y = h(x)$. We have included a number of points in addition to the asymptotes:



The dots have been placed at the following points on the graph:

$(-8, 22/9) \quad (-7, 2.5) \quad (-6, 18/7) \quad (-5, 8/3) \quad (-4, 2.8)$

$(-3, 3) \quad (-2, 10/3) \quad (-1, 4) \quad (0, 6) \quad (0.2, 7)$

$(1.4, -8) \quad (1.5, -6) \quad (2, -2) \quad (3, 0)$

$(4, 2/3) \quad (5, 1) \quad (6, 1.2) \quad (7, 4/3) \quad (8, 10/7)$